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Obtaining CKM phase information from B penguin decays

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Abstract: We discuss a method for extracting CP phases from pairs of B decays which are related by flavor SU(3). One decay ($B^0 \to M_1 M_2$) receives a significant $\bar{b} \to \bar{d}$ penguin contribution. The second ($B' \to M_1' M_2'$) has a significant $\bar{b} \to \bar{s}$ penguin contribution, but is dominated by a single amplitude. CP phase information is obtained using the fact that the $B' \to M_1' M_2'$ amplitude is related by SU(3) to a piece of the $B^0 \to M_1 M_2$ amplitude. The leading-order SU(3)-breaking effect ($\sim 25\%$) responsible for the main theoretical error can be removed. For some decay pairs, it can be written in terms of known decay constants. In other cases, it involves a ratio of form factors. However, this form-factor ratio can either be measured experimentally, or eliminated by considering a double ratio of amplitudes. In all cases, one is left only with a second-order effect, $\sim 5\%$. We find twelve pairs of B decays to which this method can be applied. Depending on the pair, we estimate the total theoretical error in relating the $B' \to M_1' M_2'$ and $B^0 \to M_1 M_2$ amplitudes to be between 5% and 15%. The most promising decay pairs are $B_d^0 \to \pi^+\pi^-$ and $B_u^+ \to K^0\pi^+$, and $B_d^0 \to D^+D^-$ and $B_d^0 \to D_s^+ D^-$ or $B_u^+ \to D_s^+ \bar{D}^0$.

Keywords: Weak Decays, B-Physics, CP violation.

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Contents

1. Introduction

Within the standard model (SM), CP violation arises because of a complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. This phase information can be encoded in the so-called unitarity triangle, whose interior angles are known as α, β and γ [1]. The independent measurement of each of these CP angles will overconstrain the unitarity triangle, thereby testing the SM.

One can perform further tests of the SM by measuring these angles in many different ways. With this in mind, numerous methods, usually involving B decays, have been proposed for measuring the CP phases [2]. In general, these techniques suffer from some degree of theoretical error due to the hadronic uncertainty in B decays. Some methods are quite clean, i.e. they have little theoretical uncertainty (e.g. the extraction of β in $B_d^0(t) \to J/\psi K_S$), while others have considerable hadronic uncertainty. Obviously, the most interesting strategies are those which have as small a theoretical error as possible.

One class of methods relies on flavor SU(3) symmetry [3]. Under this symmetry, the quarks u, d and s all are placed in the same multiplet. As a result, many particles are related by SU(3),[1] including π's and K's, B_d^0 and B_s^0 mesons, and D and D_s mesons. Many techniques have been proposed to obtain weak phase information from decays related by SU(3). Unfortunately, all of these methods suffer from hadronic uncertainties due to the breaking of the SU(3) symmetry, typically $O(m_s/\Lambda_{QCD}) \sim 25\%$. In order to obtain precise CP phase information from a given method, this theoretical error must be reduced. For example, in some techniques, the leading-order SU(3) breaking can be cast as the ratio of (measured) decay constants, leaving a theoretical uncertainty at the level of second-order SU(3) breaking.

Recently, we proposed two new methods, based on SU(3), for obtaining CP phases. The measurements of $B_{d,s}^0 \to K^{(*)}\bar{K}^{(*)}$ yield the angle α [4], while γ can be extracted from $B_d^0(t) \to D^{(*)+}D^{(*)-}$ and $B_d^0 \to D_s^{(*)+}D^{(*)-}$ [5]. In addition, Gronau and Rosner have noted that either α or γ can be obtained from $B_d^0(t) \to \pi^+\pi^-$ and $B_u^+ \to K^0\pi^+$ [6] (and they performed the analysis using the available data). In all cases, it is argued that the leading-order SU(3) breaking is under control. In ref. [4] this is because a double ratio is used, so that the leading-order SU(3)-breaking effect cancels. In ref. [5], SU(3) breaking is due mainly to the ratio of decay constants, f_{D_s}/f_D, which is known quite precisely in lattice gauge theory. (If one is reluctant to use this input, one can instead use a double ratio, as in ref. [4].) The upshot is that the remaining theoretical error is at the level of a second-order effect, $\sim 5\%$. Finally, in ref. [6], the leading-order SU(3)-breaking effect is given by the (known) ratio of decay constants f_K/f_π.

The purpose of the present paper is to further explore these techniques. As we will show, they are all essentially the same method. Furthermore, one can apply this method to other pairs of B decays — the size of the theoretical error depends on the particular decays considered.

The basic idea of the method is the following. The amplitude for a $B^0 \to M_1M_2$ decay with a significant $\bar{b} \to \bar{d}$ penguin contribution can be written $A_uV_{ub}^*V_{ud} + A_cV_{cb}^*V_{cd} + A_tV_{tb}^*V_{td} = (A_u - A_t)V_{ub}^*V_{ud} + (A_c - A_t)V_{cb}^*V_{cd}$, where we have used the unitarity of the CKM matrix to eliminate the $V_{tb}^*V_{td}$ term. It has been shown that one cannot obtain clean CKM phase information from the measurement of such a decay — one always needs (at least) one piece of theoretical input [7]. Now consider a second decay $B' \to M_1'M_2'$ which receives a significant $\bar{b} \to \bar{s}$ penguin contribution. The amplitude for this decay can be

[1]Some methods refer to U-spin symmetry, which interchanges d and s quarks. U-spin is a subgroup of the full SU(3) symmetry.

written $(A'_u - A'_t)V^*_{ub}V_{us} + (A'_c - A'_t)V^*_{cb}V_{cs} \approx (A'_c - A'_t)V^*_{cb}V_{cs}$. Here we have used the fact that $|V^*_{ub}V_{us}| \ll |V^*_{cb}V_{cs}|$. We now make the theoretical assumption that $(A'_c - A'_t)$ can be related to $(A_c - A_t)$ by flavor SU(3). In this case, the measurement of the branching ratio for $B' \to M'_1 M'_2$ will give us the necessary input to extract CKM phase information from measurements of the first decay. There are two sources of theoretical uncertainty in such a method. First, $(A'_c - A'_t)$ and $(A_c - A_t)$ may not be exactly equal in the SU(3) limit. And second, there will necessarily be some SU(3) breaking in relating the two amplitudes. In order to minimize this theoretical error, one must choose the two decays carefully.

The paper is organized as follows. In section 2, we describe the method in considerably more detail. We show that, because of CKM unitarity, one can extract either α or γ. The next two sections are somewhat more technical. The reader who wishes to skip these details can move directly to section 5. In section 3 we examine which pairs of decays are useful for this method. We choose decay pairs for which $(A'_c - A'_t)$ and $(A_c - A_t)$ are exactly related in the SU(3) limit or are related by SU(3) provided certain small amplitudes can be neglected. We find twelve such pairs. The next step is to estimate the SU(3) breaking for each of these pairs. This is done in section 4 using large N_c QCD and QCD factorization [8]. We summarize and discuss our findings in section 5. In all cases, although the size of SU(3) breaking is about 25%, we argue that the theoretical uncertainty can be reduced to the level of a second-order effect, $\sim 5\%$. In some cases, this comes about because the leading-order SU(3) breaking is given by a (known) ratio of decay constants. In the other cases, the SU(3) breaking involves an unknown ratio of form factors. We show that, for many decay pairs, this ratio can be measured. Failing this, one can reduce the SU(3) theoretical uncertainty by using a double ratio of amplitudes. Depending on the pair of B decays chosen, and how one combines the various theoretical uncertainties, the total theoretical error in relating the $B' \to M'_1 M'_2$ and $B^0 \to M_1 M_2$ amplitudes is between 5% and 15%. We find that our method is most promising for three decay pairs: $B^0_d \to \pi^+\pi^-$ and $B^+_u \to K^0\pi^+$ [6], for which data is already available, and $B^0_d \to D^+D^-$ and $B^0_d \to D^+_s D^-$ [5] or $B^+_u \to D^+_s \bar{D}^0$. In section 6, using the latest data, we update ref. [6], and show how γ can be obtained from measurements of the decay pair $B^0_d \to \pi^+\pi^-$ and $B^+_u \to K^0\pi^+$. We conclude in section 7.

2. Extraction of CP phases: general case

In this section, we describe the method for obtaining CP phase information in as general terms as possible. We do not refer to specific decays here; these are studied in the next section.

2.1 Method I

Consider a neutral $B^0 \to M_1 M_2$ decay involving a $\bar{b} \to \bar{d}$ penguin amplitude. B^0 can be either a B^0_d or a B^0_s meson, and M_1 and M_2 are two mesons. (If both M_1 and M_2 are vector mesons, the final state can be considered as a single helicity state of $M_1 M_2$.) The decay $B^0 \to M_1 M_2$ can be a pure penguin decay, or can involve both tree and penguin contributions. In the latter case, it is assumed that the penguin amplitude is not negligible.

In addition, we only consider final states M_1M_2 accessible to both B^0 and $\overline{B}^0$ mesons. Thus, one expects indirect CP violation in such decays.

The general amplitude for $B^0 \to M_1M_2$ can be written

$$\begin{aligned} A(B^0 \to M_1M_2) &= A_u V_{ub}^* V_{ud} + A_c V_{cb}^* V_{cd} + A_t V_{tb}^* V_{td} \\ &= (A_u - A_t) V_{ub}^* V_{ud} + (A_c - A_t) V_{cb}^* V_{cd} \\ &\equiv \mathcal{A}_{ut}\, e^{i\gamma} e^{i\delta_{ut}} + \mathcal{A}_{ct}\, e^{i\delta_{ct}} , \end{aligned} \tag{2.1}$$

where $\mathcal{A}_{ut} \equiv |(A_u - A_t)V_{ub}^* V_{ud}|$, $\mathcal{A}_{ct} \equiv |(A_c - A_t)V_{cb}^* V_{cd}|$, and we have explicitly written the strong phases δ_{ut} and δ_{ct}, as well as the weak phase γ. In passing from the first line to the second, we have used the unitarity of the CKM matrix, $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$, to eliminate the $V_{tb}^* V_{td}$ term.

The amplitude $\overline{A}$ describing the conjugate decay $\overline{B}^0 \to \overline{M}_1\overline{M}_2$ can be obtained from eq. (2.1) by changing the sign of γ. Below we assume that M_1M_2 is self-conjugate, $\overline{M}_1\overline{M}_2 = M_1M_2$. We consider the case in which $\overline{M}_1\overline{M}_2 \neq M_1M_2$ in section 2.3.

The time-dependent measurement of $B^0(t) \to M_1M_2$ allows one to obtain the three observables

$$\begin{aligned} B &\equiv \frac{1}{2}\left(|A|^2 + |\overline{A}|^2\right) = \mathcal{A}_{ct}^2 + \mathcal{A}_{ut}^2 + 2\mathcal{A}_{ct}\,\mathcal{A}_{ut}\cos\delta\cos\gamma\,, \\ a_{dir} &\equiv \frac{1}{2}\left(|A|^2 - |\overline{A}|^2\right) = -2\mathcal{A}_{ct}\,\mathcal{A}_{ut}\sin\delta\sin\gamma\,, \\ a_I &\equiv \mathrm{Im}\left(e^{-2i\beta} A^* \overline{A}\right) = -\mathcal{A}_{ct}^2 \sin 2\beta - 2\mathcal{A}_{ct}\,\mathcal{A}_{ut}\cos\delta\sin(2\beta+\gamma) - \mathcal{A}_{ut}^2\sin(2\beta+2\gamma)\,, \end{aligned} \tag{2.2}$$

where $\delta \equiv \delta^{ut} - \delta^{ct}$. It is useful to define a fourth observable:

$$a_R \equiv \mathrm{Re}\left(e^{-2i\beta} A^* \overline{A}\right) = \mathcal{A}_{ct}^2 \cos 2\beta + 2\mathcal{A}_{ct}\,\mathcal{A}_{ut}\cos\delta\cos(2\beta+\gamma) + \mathcal{A}_{ut}^2\cos(2\beta+2\gamma)\,. \tag{2.3}$$

The quantity a_R is not independent of the other three observables:

$$a_R^2 = B^2 - a_{dir}^2 - a_I^2\,. \tag{2.4}$$

Thus, one can obtain a_R from measurements of B, a_{dir} and a_I, up to a sign ambiguity.

The three independent observables depend on five theoretical parameters: $\mathcal{A}_{ut}$, $\mathcal{A}_{ct}$, δ, β, γ. Therefore one cannot obtain CP phase information from these measurements [7]. However, one can partially solve the equations to obtain

$$\mathcal{A}_{ct}^2 = \frac{a_R\cos(2\beta+2\gamma) - a_I\sin(2\beta+2\gamma) - B}{\cos 2\gamma - 1}\,. \tag{2.5}$$

Thus, assuming that 2β is known from the measurement of CP violation in $B_d^0(t) \to J/\psi K_S$, we could obtain γ if we knew the value of $\mathcal{A}_{ct}$.

Consider now a decay $B' \to M_1'M_2'$ involving a $\bar{b} \to \bar{s}$ penguin. We refer to this as the "partner process." This decay is related by SU(3) symmetry to $B^0 \to M_1M_2$. (In section 3, for a given $B^0 \to M_1M_2$ decay, we present a variety of possibilities for the partner process.)

In this case, B' (B_d^0, B_s^0 or B_u^+) and $M_1' M_2'$ depend on the choice of the partner process. The amplitude for $B' \to M_1' M_2'$ can be written

$$\begin{aligned} A(B' \to M_1' M_2') &= A_u' V_{ub}^* V_{us} + A_c' V_{cb}^* V_{cs} + A_t' V_{tb}^* V_{ts} \\ &= \mathcal{A}_{ut}' \, e^{i\gamma} e^{i\delta_{ut}'} + \mathcal{A}_{ct}' \, e^{i\delta_{ct}'} \\ &\approx \mathcal{A}_{ct}' \, e^{i\delta_{ct}'} \,, \end{aligned} \tag{2.6}$$

where $\mathcal{A}_{ut}' \equiv |(A_u' - A_t') V_{ub}^* V_{us}|$ and $\mathcal{A}_{ct}' \equiv |(A_c' - A_t') V_{cb}^* V_{cs}|$.

Note that $V_{ub}^* V_{us}$ is much smaller than $V_{cb}^* V_{cs}$: $|V_{ub}^* V_{us} / V_{cb}^* V_{cs}| \simeq 2\%$. The assumption that $(A_u' - A_t')$ is not greatly enhanced relative to $(A_c' - A_t')$ implies that $\mathcal{A}_{ut}' \ll \mathcal{A}_{ct}'$, leading to the last line above. That is, *the partner process is assumed to be dominated by a single amplitude.* There are several ways of testing this assumption experimentally, such as the direct CP asymmetry in $B_u^\pm \to \pi^\pm K$ and $BR(B_u^\pm \to K^\pm K)$. However, at present the data do not indicate any anomalous behaviour [16]. This assumption therefore contributes only a small theoretical error, at the percent level, and the measurement of the rate for $B' \to M_1' M_2'$ yields $\mathcal{A}_{ct}'$.

We now make the SU(3) assumption that

$$\frac{\lambda \mathcal{A}_{ct}'}{\mathcal{A}_{ct}} = 1 \,, \tag{2.7}$$

where $\lambda = 0.22$ is the Cabibbo angle. Combined with the relation in eq. (2.5), this allows one to extract γ.

The theoretical uncertainty in this method is essentially given by the degree to which eq. (2.7) is violated. This can occur in two ways. First, even in the SU(3) limit, one might have $\lambda \mathcal{A}_{ct}' / \mathcal{A}_{ct} \neq 1$. We will only consider pairs of decays for which this error is small, at most $\sim 5\%$. Second, there are SU(3)-breaking effects in eq. (2.7). For a given set of decays, the size of this error can be estimated — we expect it to be of first order in SU(3) breaking, $O(m_s/\Lambda_{QCD}) \sim 25\%$. As we will see, for certain pairs of B decays, this breaking can be expressed in terms of a ratio of decay constants. If these decay constants are known, then the leading-order SU(3) breaking is under control, leaving an unknown second-order effect of $\sim 5\%$. In this case, the assumption of eq. (2.7) allows one to obtain CKM phase information with a reasonably small theoretical error. (This is the method described to obtain γ from $B_d^0(t) \to D^{(*)+} D^{(*)-}$ and $B_d^0 \to D_s^{(*)+} D^{(*)-}$ [5].) In what follows, we will refer to this as Method I.

There is an alternative, equivalent way to describe this method. Consider again the amplitude for $B^0 \to M_1 M_2$ (eq. (2.1)). If CKM unitarity is used to instead eliminate the $V_{cb}^* V_{cd}$ term, one has

$$A(B^0 \to M_1 M_2) = \mathcal{A}_{uc} \, e^{i\gamma} e^{i\delta_{uc}} + \mathcal{A}_{tc} \; e^{-i\beta} e^{i\delta_{tc}} \,, \tag{2.8}$$

where $\mathcal{A}_{uc} \equiv |(A_u - A_c) V_{ub}^* V_{ud}|$, $\mathcal{A}_{tc} \equiv |(A_t - A_c) V_{tb}^* V_{td}|$. In this parametrization the three independent observables measured in $B^0(t) \to M_1 M_2$ depend on four theoretical parameters: $\mathcal{A}_{uc}$, $\mathcal{A}_{tc}$, $\Delta \equiv \delta^{uc} - \delta^{tc}$, α. It is therefore still not possible to obtain CP phase information from these measurements. However, one can express

$$\mathcal{A}_{tc}^2 = \frac{a_R \cos 2\alpha + a_I \sin 2\alpha - B}{\cos 2\alpha - 1} . \tag{2.9}$$

If we knew $\mathcal{A}_{tc}$ we could extract α.

Using the logic described above, we consider the partner decay $B' \to M_1' M_2'$. When the $V_{cb}^* V_{cd}$ term in eq. (2.6) is eliminated, one obtains

$$A(B' \to M_1' M_2') \approx \mathcal{A}'_{tc}\, e^{i\delta'_{tc}} , \tag{2.10}$$

where $\mathcal{A}'_{tc} \equiv |(A'_t - A'_c) V_{tb}^* V_{ts}|$. The measurement of the branching ratio for $B' \to M_1' M_2'$ therefore yields $\mathcal{A}'_{tc}$.

Now the SU(3) relation between $\mathcal{A}_{tc}$ and $\mathcal{A}'_{tc}$ is [6]

$$\frac{\lambda \mathcal{A}'_{tc}}{\mathcal{A}_{tc}} = \lambda \left| \frac{V_{ts}}{V_{td}} \right| = \frac{\sin\alpha}{\sin\gamma} . \tag{2.11}$$

Writing $\gamma = \pi - \alpha - \beta$, and assuming that β has been measured, one can extract α from eq. (2.9).

Based on the above discussion, it appears that, depending on which parametrization of the amplitudes is used, one can extract either γ or α. However, these are equivalent. In both cases, we assume that β is known. Since $\alpha + \beta + \gamma = \pi$, knowledge of one of α or γ allows one to derive the other angle. This simply reflects the fact that the three CP phases are not independent. (In ref. [7], this is referred to as the "CKM ambiguity.")

2.2 Method II

One can remove the leading-order SU(3)-breaking effect as follows. Consider a second decay $B^0 \to \widetilde{M}_1 \widetilde{M}_2$, where $\widetilde{M}_{1,2}$ are either excited states, or different helicity states, of $M_{1,2}$. The amplitude for $B^0 \to \widetilde{M}_1 \widetilde{M}_2$ is given by an expression analogous to eq. (2.1):

$$A(B^0 \to \widetilde{M}_1 \widetilde{M}_2) = \widetilde{\mathcal{A}}_{ut} e^{i\gamma} e^{i\tilde{\delta}_{ut}} + \widetilde{\mathcal{A}}_{ct} e^{i\tilde{\delta}_{ct}} . \tag{2.12}$$

The time-dependent measurement of $B^0(t) \to \widetilde{M}_1 \widetilde{M}_2$ allows one to obtain $\widetilde{a}_R$, $\widetilde{a}_I$ and $\widetilde{B}$, analogous to the observables in $B^0(t) \to M_1 M_2$. We then have

$$\frac{\mathcal{A}_{ct}^2}{\widetilde{\mathcal{A}}_{ct}^2} = \frac{a_R \cos(2\beta + 2\gamma) - a_I \sin(2\beta + 2\gamma) - B}{\widetilde{a}_R \cos(2\beta + 2\gamma) - \widetilde{a}_I \sin(2\beta + 2\gamma) - \widetilde{B}} . \tag{2.13}$$

As before, given an independent measurement of 2β, the knowledge of $\mathcal{A}_{ct}/\widetilde{\mathcal{A}}_{ct}$ would allow us to obtain γ.

This information can be obtained by considering a second partner process, $B' \to \widetilde{M}_1' \widetilde{M}_2'$, where $\widetilde{M}_{1,2}'$ are either excited states, or different helicity states, of $M_{1,2}'$. Analogous to eq. (2.6), we have

$$A(B' \to \widetilde{M}_1' \widetilde{M}_2') \approx \widetilde{\mathcal{A}}'_{ct}\, e^{i\tilde{\delta}'_{ct}} . \tag{2.14}$$

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The measurement of the rate for $B' \to \widetilde{M}_1'\widetilde{M}_2'$ yields $\widetilde{\mathcal{A}}_{ct}'$. Now the assumption that

$$\frac{\mathcal{A}_{ct}'/\widetilde{\mathcal{A}}_{ct}'}{\mathcal{A}_{ct}/\widetilde{\mathcal{A}}_{ct}} = 1 \tag{2.15}$$

provides the information necessary to obtain γ from eq. (2.13).

Because we rely on a double ratio, we expect a significant cancellation of the SU(3)-breaking effects. For example, this occurs in decays where the leading-order SU(3) breaking is expressible in terms of decay constants. In this case, the decay constants cancel in eq. (2.15) for particular pairs of processes, leaving only a second-order correction of $\sim 5\%$. In the more general case, where SU(3) breaking involves also form factors, there is no *proof* that the leading-order SU(3)-breaking effect cancels in the double ratio. However, it is intuitively reasonable and this cancellation, under certain conditions, can be demonstrated for particular final states. In general significant cancellation of SU(3) breaking effects in ratios of form factors are found in all explicit calculations.

The theoretical error in eq. (2.15) ($\sim 5\%$) is therefore considerably smaller than that of eq. (2.7) ($\sim 25\%$). For this reason it is often more advantageous to use the method with the double ratio. (For specific decays, we provide quantitative estimates of these theoretical uncertainties in section 4.) Henceforth, we will refer to this as Method II.

If one uses the parametrization of eq. (2.8), then the relation analogous to eq. (2.13) is

$$\frac{\mathcal{A}_{tc}^2}{\widetilde{\mathcal{A}}_{tc}^2} = \frac{a_R \cos 2\alpha + a_I \sin 2\alpha - B}{\widetilde{a}_R \cos 2\alpha + \widetilde{a}_I \sin 2\alpha - \widetilde{B}}\,. \tag{2.16}$$

(Note that $\mathcal{A}_{ct}^2/\widetilde{\mathcal{A}}_{ct}^2 = \mathcal{A}_{tc}^2/\widetilde{\mathcal{A}}_{tc}^2$, so that eq. (2.16) is actually *identical* to eq. (2.13).) In order to obtain the ratio on the left-hand side, we once again consider the second partner decay, $B' \to \widetilde{M}_1'\widetilde{M}_2'$:

$$A(B' \to \widetilde{M}_1'\widetilde{M}_2') \approx \widetilde{\mathcal{A}}_{tc}'\, e^{i\tilde{\delta}_{tc}'}\,. \tag{2.17}$$

The measurement of the rate for $B' \to \widetilde{M}_1'\widetilde{M}_2'$ yields $\widetilde{\mathcal{A}}_{tc}'$. One then makes the assumption that

$$\frac{\mathcal{A}_{tc}'/\widetilde{\mathcal{A}}_{tc}'}{\mathcal{A}_{tc}/\widetilde{\mathcal{A}}_{tc}} = 1\,. \tag{2.18}$$

In the case the factor of $\sin\alpha/\sin\gamma$ in eq. (2.11) cancels between numerator and denominator. Thus, Method II allows one to extract α from eq. (2.16) *without* assuming knowledge of β. (This is the method described in ref. [4].)

2.3 Method II$'$

Finally, we briefly discuss the case in which the final state in the decay $B^0 \to M_1M_2$ is not self-conjugate: $\overline{M}_1\overline{M}_2 \neq M_1M_2$. We describe the method for the parametrization of eq. (2.8). As we will see below, the CP phase α can be extracted. This is equivalent to the method in which γ is obtained, assuming that β is known.

One now must consider separately the two decays $B^0(t) \to M_1M_2$ and $B^0(t) \to \overline{M}_1\overline{M}_2$:

$$\begin{aligned} A(B^0 \to M_1M_2) &= \mathcal{A}_{uc}\, e^{i\gamma} e^{i\delta_{uc}} + \mathcal{A}_{tc}\; e^{-i\beta} e^{i\delta_{tc}}\,, \\ A(B^0 \to \overline{M}_1\overline{M}_2) &= \widetilde{\mathcal{A}}_{ut}\, e^{i\gamma} e^{i\tilde{\delta}_{ut}} + \widetilde{\mathcal{A}}_{tc}\, e^{-i\beta} e^{i\tilde{\delta}_{tc}}\,. \end{aligned} \tag{2.19}$$

As before, one can extract the following observables from the time-dependent measurements of these decays: B, a_{dir}, a_I, a_R, $\widetilde{B}$, $\tilde{a}_{dir}$, $\tilde{a}_I$ and $\tilde{a}_R$. With a bit of algebra, one can derive the following expression [4]:

$$W = X \tan 2\alpha + \frac{\widetilde{\mathcal{A}}_{tc}}{\mathcal{A}_{tc}} \frac{Y}{2\cos 2\alpha} - \frac{\mathcal{A}_{tc}}{\widetilde{\mathcal{A}}_{tc}} \frac{Z}{2\cos 2\alpha}\,, \tag{2.20}$$

where

$$\begin{aligned} W &\equiv \frac{1}{2}(a_R - \tilde{a}_R)\,, \\ X &\equiv \frac{1}{2}(-a_I + \tilde{a}_I)\,, \\ Y &\equiv \frac{1}{2}(-B - a_{dir} + \widetilde{B} - \tilde{a}_{dir})\,, \\ Z &\equiv \frac{1}{2}(B - a_{dir} - \widetilde{B} - \tilde{a}_{dir})\,. \end{aligned} \tag{2.21}$$

If we knew $\mathcal{A}_{tc}/\widetilde{\mathcal{A}}_{tc}$ we could extract α.

This information can be obtained from the partner processes $B' \to M_1' M_2'$ and $B' \to \overline{M}_1' \overline{M}_2'$ which are each dominated by a single decay amplitude. The measurement of the rates for these decays allows one to extract $\mathcal{A}_{tc}'$ and $\widetilde{\mathcal{A}}_{tc}'$. We assume that

$$\frac{\mathcal{A}_{tc}'/\widetilde{\mathcal{A}}_{tc}'}{\mathcal{A}_{tc}/\widetilde{\mathcal{A}}_{tc}} = 1\,. \tag{2.22}$$

This provides the theoretical input necessary to obtain α from eq. (2.20). As before, the leading-order SU(3)-breaking effect cancels in the double ratio, so that the net theoretical error is a second-order effect.

We have therefore shown that it is possible to extract CP phase information from time-dependent measurements of the decays $B^0(t) \to M_1 M_2$ and $B^0(t) \to \widetilde{M}_1 \widetilde{M}_2$, along with rate measurements of their SU(3)-related partner processes $B' \to M_1' M_2'$ and $B' \to \widetilde{M}_1' \widetilde{M}_2'$. The above discussion has been completely general; in the next section we turn to an examination of the specific decays to which this method can be applied.

3. Specific decays

Here we examine the decays to which the method described in the previous section can be applied. The first step is to find neutral $B^0 \to M_1 M_2$ decays involving a $\bar{b} \to \bar{d}$ penguin amplitude, with the condition that both B^0 and $\overline{B}^0$ mesons can decay to $M_1 M_2$. Such decays are straightforward to tabulate. They are:

$$\begin{aligned} B_d^0 &\to D^+ D^-\,, \qquad \pi^+\pi^-\,, \qquad \pi^0\pi^0\,, \qquad K^0\bar{K}^0\,, \\ B_s^0 &\to \bar{K}^0\pi^0\,, \qquad \eta_s \bar{K}^0\,. \end{aligned} \tag{3.1}$$

In all cases the final state is written in terms of pseudoscalars (P's) only. However, it is understood that either or both of the final-state particles can be vector mesons (V's). In the case of two vector mesons, there are three helicity states; each of these can be considered as a separate final state.

Recall that we require either that the final state be a CP eigenstate (Methods I and II), or that the B^0 be able to decay to both M_1M_2 and $\overline{M}_1\overline{M}_2$ (Method II$'$). As written, the B_s^0 decays above do not satisfy either of these conditions because of the presence of the '$\bar{K}^0$' in the final state. (As noted above, this is to be understood as either a P or a V.) In order to apply our methods to these decays, this particle must be a K_S if it is a pseudoscalar. If the final-state particle is a $\bar{K}^{*0}$, it must be detected via its decay to $K_S\pi^0$.

In eq. (3.1), η_s corresponds to a pure $(s\bar{s})$ quark pair. In practice, if the $(s\bar{s})$ hadronizes as a pseudoscalar, it will be found as either an η or η' meson, both of which also have significant $(u\bar{u})$ and $(d\bar{d})$ components [9]. As a result, in decays involving an η or η', $B^0 \to M_1M_2$ and $B' \to M_1'M_2'$ are not really related by SU(3), and our method does not apply. It is therefore better to consider the vector meson ϕ which is a pure $(s\bar{s})$ quark state to a very good approximation.

The second step is to find partner processes $B' \to M_1'M_2'$, related to $B^0 \to M_1M_2$ by SU(3) symmetry, which involve a $\bar{b} \to \bar{s}$ penguin amplitude. This is more complicated, as there are several possibilities.

Consider first the decay $B_d^0 \to D^+D^-$, which receives contributions from a $\bar{b} \to \bar{d}c\bar{c}$ penguin amplitude. Under SU(3), the d, s and u quarks are treated on an equal footing. Therefore there are three possible candidate partner processes for this decay. The $\bar{b} \to \bar{d}$ penguin amplitude is changed to a $\bar{b} \to \bar{s}$ amplitude, and one considers three different flavors for the spectator quark:

$$B_s^0 \to D_s^+D_s^- \,, \qquad B_d^0 \to D_s^+D^- \,, \qquad B_u^+ \to D_s^+\bar{D}^0 \,. \tag{3.2}$$

The remaining five decays in eq. (3.1) all involve the decay of a $\bar{b}$ quark into light quarks. Since all three light quarks are equivalent under SU(3), the potential partner processes are given by a B_d^0, B_s^0 or B_u^+ meson decaying via a $\bar{b} \to \bar{s}$ penguin diagram, with the gluon (or Z) turning into a $\bar{u}u$, $\bar{d}d$ or $\bar{s}s$ quark pair. That is, the candidate partner processes for all five decays are

$$\begin{aligned} B_d^0 &\to K^+\pi^- \,, &\quad K^0\pi^0 \,, &\quad \eta_s K^0 \,, \\ B_s^0 &\to K^+K^- \,, &\quad K^0\bar{K}^0 \,, &\quad \eta_s\eta_s \,, \\ B_u^+ &\to K^+\pi^0 \,, &\quad K^0\pi^+ \,, &\quad \eta_s K^+ \,. \end{aligned} \tag{3.3}$$

Now, recall that one of the requirements of the method is that the partner process be dominated by a single amplitude. Unfortunately, the decays $B_d^0 \to K^+\pi^-$, $B_s^0 \to K^+K^-$ and $B_u^+ \to K^+\pi^0$ above receive significant contributions from both tree and $\bar{b} \to \bar{s}$ penguin amplitudes. Thus, they are not dominated by a single amplitude, and do not satisfy the above requirement. Therefore, they cannot be used as partner processes in our method.

(On the other hand, we note that ref. [10] uses U-spin symmetry to relate $B_d^0 \to \pi^+\pi^-$ to $B_s^0 \to K^+K^-$ (or $B_d^0 \to K^+\pi^-$, if exchange-type diagrams are neglected). Here it is assumed that direct CP violation is measured in the partner process. With the assumption of U-spin symmetry, one can extract γ. It has been argued that the SU(3)-breaking corrections in this approach may be sizeable [11]. However, this occurs if annihilation contributions are large, which is not the naive expectation. We discuss the SU(3)-breaking effects in such contributions in section 4.7.)

Most pairs of decays ($B^0 \to M_1 M_2$ and its partner process) have not been studied, but there are a few exceptions. As already noted, we have examined $B_d^0 \to K^0 \bar{K}^0$ and $B_s^0 \to K^0 \bar{K}^0$ [4], as well as $B_d^0 \to D^+ D^-$ and $B_d^0 \to D_s^+ D^-$ [5]. In addition, Fleischer has noted that one can get γ from $B_d^0 \to D^+ D^-$ and $B_s^0 \to D_s^+ D_s^-$ decays [12]. In this paper one keeps both contributing amplitudes to $B_s^0 \to D_s^+ D_s^-$, and assumes that their (small) interference is measured. By using U-spin to relate $B_d^0 \to D^+ D^-$ and $B_s^0 \to D_s^+ D_s^-$, one can obtain γ. However, if one neglects the small $V_{ub}^* V_{us}$ amplitude in $B_s^0 \to D_s^+ D_s^-$, one essentially reproduces the method outlined in section 2. Finally, the pair $B_d^0 \to \pi^+ \pi^-$ and $B_u^+ \to K^0 \pi^+$ has been studied in ref. [6].

In the remainder of this section, we will examine all $B^0 \to M_1 M_2$ decays, along with their potential partner processes, to see which can be used to obtain CP phase information with our method. At this stage the goal is simply to find pairs of processes in which the amplitudes $\mathcal{A}_{ct}$ and $\lambda \mathcal{A}'_{ct}$ (or, equivalently, $\mathcal{A}_{tc}$ and $\lambda \mathcal{A}'_{tc}$) are equal in the SU(3) limit; SU(3) breaking will be studied in the next section.

Within SU(3), all $B^0 \to M_1 M_2$ decays can be expressed in terms of a small number of matrix elements. This is equivalent to a description in terms of diagrams [13, 14]. Including electroweak penguin contributions (EWP's), there are eight main contributing diagrams: (1) a color-favored tree amplitude T, (2) a color-suppressed tree amplitude C, (3) a gluonic penguin amplitude P, (4) an exchange amplitude E, (5) an annihilation amplitude A, (6) a penguin annihilation amplitude PA, (7) a color-favored electroweak penguin amplitude P_{EW}, and (8) a color-suppressed electroweak penguin amplitude P_{EW}^C. For $\bar{b} \to \bar{d} q \bar{q}$ and $\bar{b} \to \bar{s} q \bar{q}$ ($q = u,\ d,\ s$), we write the diagrams with no primes and primes, respectively. For $\bar{b} \to \bar{d} c \bar{c}$ and $\bar{b} \to \bar{s} c \bar{c}$, we write the diagrams with tildes and tildes plus primes, respectively. We will express all amplitudes in terms of these diagrams.

Pairs of decays whose amplitudes are the same, except for primes, are equal in the SU(3) limit. Any difference in the amplitudes for a pair of decays will lead to a theoretical error. To estimate this error, it is important to know the relative sizes of the various diagrams. A useful rough guide is given in ref. [14]. For $\bar{b} \to \bar{d} q \bar{q}$ processes ($q = u,\ d,\ s$), the diagrams obey the following (approximate) hierarchy:

$$\begin{aligned} 1 :&\ \ |T|\,, \\ \mathcal{O}(\bar{\lambda}) :&\ \ |C|\,, \qquad |P|\,, \\ \mathcal{O}(\bar{\lambda}^2) :&\ \ |E|\,, \qquad |A|\,, \qquad |P_{EW}|\,, \\ \mathcal{O}(\bar{\lambda}^3) :&\ \ |PA|\,, \qquad |P_{EW}^C|\,, \end{aligned} \tag{3.4}$$

while for $\bar{b} \to \bar{s} q \bar{q}$ transitions ($q = u,\ d,\ s$), one has

$$\begin{aligned} 1 :&\ \ |P'|\,, \\ \mathcal{O}(\bar{\lambda}) :&\ \ |T'|\,, \qquad |P'_{EW}| \\ \mathcal{O}(\bar{\lambda}^2) :&\ \ |C'|\,, \qquad |PA'|\,, \qquad |P'^C_{EW}| \\ \mathcal{O}(\bar{\lambda}^3) :&\ \ |E'|\,, \qquad |A'|\,. \end{aligned} \tag{3.5}$$

In the above, $\bar{\lambda} \sim 20\%$. There are a variety of sources for these suppressions: (1) CKM matrix elements: e.g. $T' \sim V_{ub}^* V_{us} \sim \bar{\lambda}^4$, $P' \sim V_{cb}^* V_{cs} \sim \bar{\lambda}^2$, (2) loop factors: e.g. all penguin and electroweak penguin diagrams arise at loop level, (3) color suppression: e.g. C is smaller than T by a factor of about $\bar{\lambda}$, (4) m_t: although electroweak penguin amplitudes are smaller than penguin contributions by a factor of α/α_s, their numerical importance is enhanced by a factor of m_t^2/M_Z^2 [15], (5) exchange- and annihilation-type diagrams are suppressed by $f_B/m_B \sim \bar{\lambda}^2$. Putting all these suppression factors together, most of which are reflected in the Wilson coefficients for the various operators, one arrives at the hierarchies above. Note that we have implicitly assumed that the ratios of matrix elements do not differ significantly from unity.

In what follows, we use eqs. (3.4) and (3.5) to estimate the theoretical error inherent in our method. However, we must stress that the above diagram hierarchies are based on theoretical assumptions, and are to be taken as rough estimates only. Ultimately, we will learn the relative sizes of various diagrams through the study of data. For example, recent measurements of $B \to \pi\pi$ and $B \to K\pi$ decays suggest that the ratios $|C/T|$ and $|C'/T'|$ may be somewhat larger than the value $\bar{\lambda} \sim 20\%$ given above [16, 17]. It is therefore important to keep in mind that deviations from the estimates of eqs. (3.4) and (3.5) are possible, so that the theoretical errors will be correspondingly modified.

Note that, for both $\bar{b} \to \bar{d}$ and $\bar{b} \to \bar{s}$ transitions, the annihilation-type contributions are expected to be quite small. (Here and in the rest of the paper, the term "annihilation-type" refers to A, E, PA, PA', A' and E'.) However, in some approaches to hadronic B decays, such amplitudes may be chirally enhanced if there are pseudoscalars in the final state [8, 18]. On the other hand, such chiral enhancements are not present for vector-vector final states. Ultimately, the size of annihilation-type diagrams is an experimental question, and can be tested by the measurement of decays such as $B_d^0 \to D_s^+ D_s^-$ and $B_d^0 \to K^+ K^-$.

The amplitude for $B_d^0 \to D^+ D^-$ is given by

$$A(B_d^0 \to D^+ D^-) = \widetilde{T} + \widetilde{E} + \widetilde{P} + \widetilde{PA} + \frac{2}{3}\widetilde{P}_{EW}^C , \tag{3.6}$$

while those of the potential partner processes are

$$\begin{aligned} A(B_s^0 \to D_s^+ D_s^-) &= \widetilde{T}' + \widetilde{E}' + \widetilde{P}' + \widetilde{PA}' + \frac{2}{3}\widetilde{P}_{EW}^{\prime C} , \\ A(B_d^0 \to D_s^+ D^-) &= \widetilde{T}' + \widetilde{P}' + \frac{2}{3}\widetilde{P}_{EW}^{\prime C} , \\ A(B_u^+ \to D_s^+ \bar{D}^0) &= \widetilde{T}' + \widetilde{P}' + \widetilde{A}' + \frac{2}{3}\widetilde{P}_{EW}^{\prime C} , \end{aligned} \tag{3.7}$$

The amplitudes for the remaining five $B^0 \to M_1 M_2$ decays in eq. (3.1) are given by

$$\begin{aligned} A(B_d^0 \to \pi^+\pi^-) &= -T - P - E - PA - \frac{2}{3}P_{EW}^C , \\ A(B_d^0 \to K^0\bar{K}^0) &= P + PA - \frac{1}{3}P_{EW}^C , \\ \sqrt{2}A(B_d^0 \to \pi^0\pi^0) &= P - C + E + PA - P_{EW} - \frac{1}{3}P_{EW}^C , \\ \sqrt{2}A(B_s^0 \to \bar{K}^0\pi^0) &= P - C - P_{EW} - \frac{1}{3}P_{EW}^C , \\ A(B_s^0 \to \eta_s\bar{K}^0) &= P - \frac{1}{3}P_{EW} - \frac{1}{3}P_{EW}^C . \end{aligned} \tag{3.8}$$

The amplitudes for the six potential partner processes are

$$\begin{aligned}
\sqrt{2}A(B_d^0 \to K^0\pi^0) &= P' - C' - P'_{EW} - \frac{1}{3}P'^C_{EW}\,, \\
A(B_d^0 \to \eta_s K^0) &= P' - \frac{1}{3}P'_{EW} - \frac{1}{3}P'^C_{EW}\,, \\
A(B_s^0 \to K^0\bar{K}^0) &= P' + PA' - \frac{1}{3}P'^C_{EW}\,, \\
A(B_s^0 \to \eta_s\eta_s) &= P' + PA' - \frac{1}{3}P'_{EW} - \frac{1}{3}P'^C_{EW}\,, \\
A(B_u^+ \to K^0\pi^+) &= P' + A' - \frac{1}{3}P'^C_{EW}\,, \\
A(B_u^+ \to \eta_s K^+) &= P' + A' - \frac{1}{3}P'_{EW} - \frac{1}{3}P'^C_{EW}\,.
\end{aligned} \tag{3.9}$$

For each of the decays in eq. (3.8), we wish to find the process(es) in eq. (3.9) whose amplitude is equal in the SU(3) limit. However, recall that we are comparing $\mathcal{A}_{ct}$ and $\lambda\mathcal{A}'_{ct}$, i.e. the pieces of the amplitudes proportional to $V_{cb}^*V_{cd}$ ($B^0 \to M_1M_2$) or $V_{cb}^*V_{cs}$ (partner process). Several pieces in the amplitudes of eq. (3.8) are proportional to $V_{ub}^*V_{ud}$: the T and E amplitudes in $B_d^0 \to \pi^+\pi^-$, the C and E amplitudes in $B_d^0 \to \pi^0\pi^0$, and the C amplitude in $B_s^0 \to \bar{K}^0\pi^0$. In comparing the amplitudes of eqs. (3.8) and (3.9), these pieces are unimportant.

Finally, we have noted that, for both the decay and its partner process, either or both of the final-state mesons can be a pseudoscalar or a vector. However, one has to be careful if PV states are used. Within QCD factorization [8], which we use to calculate SU(3) breaking in section 4, one of the final-state mesons comes directly from the decay of the B, while the other is produced from the vacuum. If the "vacuum meson" for the decay is a P, but is a V for the partner process (or vice-versa), then the two decays are *not* related by SU(3). The reason is that the two processes receive contributions from different operators. For example, those operators responsible for chiral enhancement affect the production of a P from the vacuum, but not a V. Thus, although the decay and its partner process are related by SU(3) at the quark level, they are not related at the meson level.

We consider each of the $B^0 \to M_1M_2$ decays in turn, labeling each of the subsections by the decay in eq. (3.1).

3.1 $B_d^0 \to D^+D^-$

$B_d^0 \to D^+D^-$ is unique among $B^0 \to M_1M_2$ decays in that its largest decay component is a $\bar{b} \to c\bar{c}\bar{d}$ tree amplitude $\widetilde{T}$. Referring to eqs. (3.6) and (3.7), we note that the amplitude for $B_s^0 \to D_s^+D_s^-$ is equal to that of $B_d^0 \to D^+D^-$ in the SU(3) limit. As for $B_d^0 \to D_s^+D^-$, its amplitude differs from that of $B_d^0 \to D^+D^-$ only by the diagrams $\widetilde{E}$ and $\widetilde{PA}$. A theoretical error is incurred in neglecting these contributions due to the fact that $|\widetilde{E}/\widetilde{T}| \sim 5\%$ and $|\widetilde{PA}/\widetilde{T}| \sim 1\%$. The decay $B_u^+ \to D_s^+\bar{D}^0$ is similar to $B_d^0 \to D_s^+D^-$, except that one has to neglect in addition the $\widetilde{A}'$ piece. However, since it is proportional to $V_{ub}^*V_{us}$, $\widetilde{A}'$ is expected to be tiny. Thus, all decays can be used as partner processes within our method, though there is a theoretical error of $\sim 5\%$ coming from the amplitudes if $B_d^0 \to D_s^+D^-$ [5] or $B_u^+ \to D_s^+\bar{D}^0$ are used.

3.2 $B_d^0 \to \pi^+\pi^-$

None of the amplitudes in eq. (3.9) is a perfect match for the amplitude of $B_d^0 \to \pi^+\pi^-$ (eq. (3.8)). Of the six potential partner processes, four receive contributions from P'_{EW}, which is not small: $|P'_{EW}/P'| \sim 20\%$ (eq. (3.5)). Thus, if any of these decays is used as the partner process, the theoretical error will be at least $\sim 20\%$. Note that there is a possible loophole here. The EWP contribution to three of the four potential partner processes is actually $(1/3)P'_{EW}$. In this case $(1/3)|P'_{EW}/P'| \sim 7\%$, which is not that large. Thus, these decays could perhaps be used as partner processes. However, this could easily be spoiled by a large ratio of matrix elements, and so we do not consider these decays as partner processes. However, the remaining two decays can be used as partner processes. In the SU(3) limit the difference in the amplitudes of $B_s^0 \to K^0\bar{K}^0$ and $B_d^0 \to \pi^+\pi^-$ is at the level of $|P'^C_{EW}/P'| \sim 5\%$. $B_u^+ \to K^0\pi^+$ is slightly worse: there are additional differences of $|A'/P'| \sim 1\%$ and $|PA/P| \sim 5\%$. Thus, both of these decays can be used as partner processes, incurring a small theoretical error due to amplitude differences in the SU(3) limit.

3.3 $B_d^0 \to K^0\bar{K}^0$

The decay $B_d^0 \to K^0\bar{K}^0$ has no P_{EW} component. Thus, in searching for partner processes to $B_d^0 \to K^0\bar{K}^0$, we can exclude the four processes in eq. (3.9) which receive contributions from P'_{EW} (see, however, the discussion in $B_d^0 \to \pi^+\pi^-$ above). Of the remaining two decays, only one has an amplitude which is equal to that of $B_d^0 \to K^0\bar{K}^0$ in the SU(3) limit: $B_s^0 \to K^0\bar{K}^0$. The other, $B_u^+ \to K^0\pi^+$, is a reasonable match, but not perfect. If it is used as a partner process, there is a theoretical error due to amplitude differences at the level of $|A'/P'| \sim 1\%$ and $|PA/P| \sim 5\%$.

3.4 $B_d^0 \to \pi^0\pi^0$

Although the final state is written as $\pi^0\pi^0$, it is understood that in this case the vector-vector state $\rho^0\rho^0$ must be used. The method requires the time-dependent measurement of the decay $B^0 \to M_1M_2$. However, this is virtually impossible for $B_d^0 \to \pi^0\pi^0$ since it is very difficult to find the vertex of a π^0.

The amplitude for the decay $B_d^0 \to \pi^0\pi^0$ has a P_{EW} component. Five of the processes in eq. (3.9) receive different contributions from P'_{EW}, and can therefore be excluded as partner processes. Only $B_d^0 \to K^0\pi^0$ is a good match: the amplitudes proportional to $V_{cb}^*V_{cq}$ $(q = d, s)$ are equal (apart from primes), except for the PA piece in $B_d^0 \to \pi^0\pi^0$. There is also a C' piece in $B_d^0 \to K^0\pi^0$ which is proportional to $V_{ub}^*V_{us}$. However, both PA and C' are expected to be small: referring to eqs. (3.4) and (3.5), we see that both $|PA/P|$ and $|C'/P'|$ are expected to be $\sim 5\%$. Because C' is very small, the requirement that the partner process be dominated by a single decay amplitude is satisfied. (Note that if it turns out that C' is larger than expected [16, 17], then this will increase the theoretical error associated with the partner process $B_d^0 \to K^0\pi^0$.)

We can therefore apply our method to the decays $B_d^0 \to \pi^0\pi^0$ and $B_d^0 \to K^0\pi^0$, incurring a theoretical uncertainty of $\sim 10\%$ due to the difference of the amplitudes. Since we know that the VV final state must be used for this particular pair of decays, from now on we will refer to them as $B_d^0 \to \rho^0\rho^0$ and $B_d^0 \to K^{*0}\rho^0$.

3.5 $B_s^0 \to \bar{K}^0\pi^0$

Like $B_d^0 \to \pi^0\pi^0$, the amplitude for $B_s^0 \to \bar{K}^0\pi^0$ has a P_{EW} component. Thus, the only decay in eq. (3.9) that can be considered as a partner process is $B_d^0 \to K^0\pi^0$. For the other decays, the theoretical error due to the amplitude differences is at the level of $|P_{EW}/P| \sim |P'_{EW}/P'| \sim 20\%$. For $B_d^0 \to K^0\pi^0$, the theoretical error is only $\sim 5\%$ due to the neglect of the C' amplitude. (This assumes the hierarchy of eq. (3.5). If C' turns out to be larger [16, 17], the theoretical error is correspondingly increased.)

As in $B_d^0 \to \pi^0\pi^0$ above, the final-state 'π^0' must be a ρ^0. The '$\bar{K}^0$' can in principle be either a pseudoscalar or a vector. However, if it is a P, then, as explained earlier, the decay $B_s^0 \to \bar{K}^0\rho^0$ is *not* related by SU(3) to $B_d^0 \to K^0\rho^0$. The reason is that, in $B_s^0 \to \bar{K}^0\rho^0$, the ρ^0 is produced from the vacuum, whereas in $B_d^0 \to K^0\rho^0$ it is the K^0. Since this particle is a pseudoscalar meson in one case, but a vector meson in the other, different operators are involved and the two decays are not related by SU(3). Thus, the vector-vector final states must be used for both the decay and partner process. Henceforth we refer to this pair of decays as $B_s^0 \to \bar{K}^{*0}\rho^0$ and $B_d^0 \to K^{*0}\rho^0$. Recall that we require that both B_s^0 and $\overline{B_s^0}$ decay to the same final state. In order for this to be possible, the $\bar{K}^{*0}$ must be detected via its decay to $K_S\pi^0$ (as in the measurement of $\sin 2\beta$ via $B_d^0(t) \to J/\psi K^*$).

3.6 $B_s^0 \to \eta_s\bar{K}^0$

In this case, there are three decays which can be taken as partner processes. The amplitude for $B_d^0 \to \eta_s K^0$ is equal to that of $B_s^0 \to \eta_s\bar{K}^0$ in the SU(3) limit. The amplitude for $B_u^+ \to \eta_s K^+$ differs only by $|A'/P'| \sim 1\%$. Finally, since $|PA'/P'|$ is very small (eq. (3.5)), $B_s^0 \to \eta_s\eta_s$ can also be taken as the partner process, though there is a small ($\sim 5\%$) theoretical error coming from the amplitudes. The other decays have theoretical errors of about $|P_{EW}/P| \sim |P'_{EW}/P'| \sim 20\%$ due to differences in the amplitudes.

Thus, $B_d^0 \to \eta_s K^0$, $B_u^+ \to \eta_s K^+$ and $B_s^0 \to \eta_s\eta_s$ can all be taken as partner processes to $B_s^0 \to \eta_s\bar{K}^0$. As discussed previously, the $(s\bar{s})$ state is best viewed as a vector ϕ meson in our method. In addition, as was the case for $B_s^0 \to \bar{K}^0\pi^0$, the final-state K-meson must be a $\bar{K}^{*0}$, detected via its decay to $K_S\pi^0$. From now on we will therefore refer to these decays via their VV final states.

4. SU(3) breaking

We have found twelve pairs of decays to which our method can be applied. They are

1. $B_d^0 \to D^+D^-$ and $B_s^0 \to D_s^+D_s^-$, $B_d^0 \to D_s^+D^-$, or $B_u^+ \to D_s^+\bar{D}^0$;
2. $B_d^0 \to \pi^+\pi^-$ and $B_s^0 \to K^0\bar{K}^0$ or $B_u^+ \to K^0\pi^+$;
3. $B_d^0 \to K^0\bar{K}^0$ and $B_s^0 \to K^0\bar{K}^0$ or $B_u^+ \to K^0\pi^+$;
4. $B_d^0 \to \rho^0\rho^0$ and $B_d^0 \to K^{*0}\rho^0$;
5. $B_s^0 \to \bar{K}^{*0}\rho^0$ and $B_d^0 \to K^{*0}\rho^0$;

6. $B_s^0 \to \phi \bar{K}^{*0}$ and $B_d^0 \to \phi K^{*0}$, $B_u^+ \to \phi K^{*+}$, or $B_s^0 \to \phi\phi$.

In all cases there is a theoretical error at the percent level due to the neglect of the $V_{ub}^* V_{us}$ term in the partner process. Some pairs of decays have an additional theoretical error due to neglecting certain diagrams. For all of the above decay pairs, this error is at the level of $\bar{\lambda}^2$ or less (eqs. (3.4) and (3.5)).

The main source of theoretical error is SU(3) breaking. In the following subsections, we examine the size of SU(3) breaking in each of the six classes above.

Throughout we ignore SU(3) breaking in annihilation-type amplitudes. There are two reasons for this. First, in most cases, the partner process is related to the principal decay only if the (small) diagrams A, E, PA, PA', A' and E' are neglected. In this situation, SU(3) breaking in such contributions is irrelevant. Second, even if the decay pairs are perfectly related in the SU(3) limit, including annihilation-type diagrams, there are no reliable theoretical methods for estimating the size of such contributions. As a result, the size of SU(3) breaking is also unreliable. Thus, our estimates of SU(3) breaking exclude annihilation-type amplitudes. However, we discuss these in greater detail in section 4.7.

4.1 $B_d^0 \to D^+ D^-$

There are three possible partner processes to $B_d^0 \to D^+ D^-$: $B_s^0 \to D_s^+ D_s^-$, $B_d^0 \to D_s^+ D^-$ and $B_u^+ \to D_s^+ \bar{D}^0$. We refer to the three pairs of decays, in this order, as pair "a", "b" and "c". All amplitudes are dominated by the tree contributions. The penguin and electroweak penguin diagrams are smaller than the tree diagrams: the ratios $\widetilde{P}/\widetilde{T}$, etc. are $\sim 25\%$ or smaller. Thus, SU(3) breaking originates mainly in the ratio $\widetilde{T}'/\widetilde{T}$ in eq. 2.7 — the SU(3)-breaking contribution from the penguin amplitudes is of higher order. Factorization has been used to study $B \to D^{(*)} \bar{D}^{(*)}$ decays [19], and it has been found that experiments are consistent with the factorization predictions. We will therefore use factorization to estimate the SU(3) breaking in the various decay pairs.

We begin by calculating $\widetilde{T}$ and $\widetilde{T}'$ within factorization. These are generated by two terms in the effective hamiltonian [20]:

$$H_{tree}^q = \frac{G_F}{\sqrt{2}} \left[V_{ub} V_{uq}^* (c_1 O_1^q + c_2 O_2^q) \right] , \tag{4.1}$$

where q can be either a d quark ($B_d^0 \to D^+ D^-$) or an s quark (partner process). The operators O_i^q are defined as

$$O_1^q = \bar{q}_\alpha \gamma_\mu L u_\beta \, \bar{u}_\beta \gamma^\mu L b_\alpha \, , \qquad O_2^q = \bar{q} \gamma_\mu L u \, \bar{u} \gamma^\mu L b \, , \tag{4.2}$$

where $L = 1 - \gamma_5$, and c_1 and c_2 are Wilson coefficients. Within factorization, the various tree amplitudes are given by

$$\begin{aligned}
|\widetilde{T}(B_d^0 \to D^+ D^-)| &= \frac{G_F}{\sqrt{2}} |V_{cb} V_{cd}^*| \left(\frac{c_1}{N_c} + c_2 \right) f_D F_0^{B_d^0 \to D} (m_D^2)(m_B^2 - m_D^2) \, , \\
|\widetilde{T}'(B_s^0 \to D_s^+ D_s^-)| &= \frac{G_F}{\sqrt{2}} |V_{cb} V_{cs}^*| \left(\frac{c_1}{N_c} + c_2 \right) f_{D_s} F_0^{B_s^0 \to D_s} (m_{D_s}^2)(m_{B_s}^2 - m_{D_s}^2) \, ,
\end{aligned}$$

$$
\begin{aligned}
|\widetilde{T}'(B_d^0 \to D_s^+ D^-)| &= \frac{G_F}{\sqrt{2}} |V_{cb} V_{cs}^*| \left(\frac{c_1}{N_c} + c_2 \right) f_{D_s} F_0^{B_d^0 \to D}(m_{D_s}^2)(m_B^2 - m_D^2) \,, \\
|\widetilde{T}'(B_u^+ \to D_s^+ \bar{D}^0)| &= \frac{G_F}{\sqrt{2}} |V_{cb} V_{cs}^*| \left(\frac{c_1}{N_c} + c_2 \right) f_{D_s} F_0^{B_d^0 \to D}(m_{D_s}^2)(m_B^2 - m_D^2) \,,
\end{aligned}
\tag{4.3}
$$

where N_c represents the number of colors. The form factor $F_0^{B\to M}$ for a $B \to M$ transition is defined through [21]

$$
\begin{aligned}
\langle M(p_K)|\bar{q}\gamma_\mu(1-\gamma_5)b|B(p_B)\rangle &= \left[(p_B + p_M)_\mu - \frac{m_B^2 - m_M^2}{q^2} q_\mu \right] F_1^{B\to M}(q^2) + \\
&\quad + \frac{m_B^2 - m_M^2}{q^2} q_\mu F_0^{B\to M}(q^2) \,,
\end{aligned}
\tag{4.4}
$$

where $q = p_B - p_M$.

The size of SU(3)-breaking for the three decays pairs is given by the deviation from unity of

$$
\begin{aligned}
\left(\frac{\lambda \widetilde{T}'}{\widetilde{T}} \right)^a &= \frac{f_{D_s} F_0^{B_s^0 \to D_s}(m_{D_s}^2)(m_{B_s}^2 - m_{D_s}^2)}{f_D F_0^{B\to D}(m_D^2)(m_B^2 - m_D^2)} \,, \\
\left(\frac{\lambda \widetilde{T}'}{\widetilde{T}} \right)^b &= \frac{f_{D_s} F_0^{B_d^0 \to D}(m_{D_s}^2)}{f_D F_0^{B_d^0 \to D}(m_D^2)} \approx \frac{f_{D_s}}{f_D} \,, \\
\left(\frac{\lambda \widetilde{T}'}{\widetilde{T}} \right)^c &= \frac{f_{D_s} F_0^{B_d^0 \to D}(m_{D_s}^2)}{f_D F_0^{B_d^0 \to D}(m_D^2)} \approx \frac{f_{D_s}}{f_D} \,.
\end{aligned}
\tag{4.5}
$$

Note that the form factor $F_0^{B_d^0 \to D}$ can be related to the Isgur-Wise function in the heavy quark limit. Since $F_0^{B_d^0 \to D}$ is smooth, there is negligible error in setting $F_0^{B_d^0 \to D}(m_{D_s}^2) = F_0^{B_d^0 \to D}(m_D^2)$. Thus, for pairs "$b$" and "$c$", SU(3) breaking is due almost entirely to the difference between the D and D_s decay constants within factorization.

The situation is worse for pair "a". Although the form factors $F_0^{B_s \to D_s}(m_{D_s}^2)$ and $F_0^{B_d^0 \to D}(m_D^2)$ are equal in the SU(3) limit, the SU(3) breaking can be calculated in chiral perturbation theory and may not be small [22]. Thus, it may be better to use $B_d^0 \to D_s^+ D^-$ or $B_u^+ \to D_s^+ \bar{D}^0$ as the partner process. Although there is a theoretical error, not present in $B_s^0 \to D_s^+ D_s^-$, due to neglecting small amplitudes, this may be smaller than the error due to SU(3) breaking in $F_0^{B_s \to D_s}(m_{D_s}^2)$ and $F_0^{B_d^0 \to D}(m_D^2)$.

For pairs "b" and "c", the leading SU(3)-breaking effect is the ratio of decay constants, f_{D_s}/f_D. This ratio has been calculated on the lattice: $f_{D_s}/f_D = 1.22 \pm 0.04$ [23]. Using this value, the error due to leading-order SU(3) breaking is quite small.

Although factorization has been shown to apply to $B \to D^{(*)}\bar{D}^{(*)}$ decays [19], it is also important to consider SU(3) breaking in nonfactorizable contributions. Evidence of factorization in $B \to D^{(*)}X$ can be understood as a consequence of large N_c QCD [24]: nonfactorizable corrections in $B \to D^{(*)}\bar{D}^{(*)}$ decays then arise at $O(1/N_c^2)$. SU(3) breaking in such effects is quite small: $\sim 25\%$ of $1/N_c^2$ is $\sim 3\%$. Therefore, the principal contribution to SU(3) breaking comes from factorizable contributions (eq. (4.5)).

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4.2 $B_d^0 \to \pi^+\pi^-$

The two potential partner processes to $B_d^0 \to \pi^+\pi^-$ are $B_s^0 \to K^0\bar{K}^0$ and $B_u^+ \to K^0\pi^+$. In order to estimate the size of SU(3) breaking in these pairs of decays, we must calculate matrix elements for nonleptonic B decays. We use QCD factorization [8] to do this. In this framework, all amplitudes are calculated in the $m_b \to \infty$ limit. The corrections are then $O(1/m_b)$. In this subsection, we present this in some detail; the description in subsequent subsections is more cursory.

The starting point for the calculation of hadronic B matrix elements is the SM effective hamiltonian for B decays [20]:

$$H_{eff}^q = \frac{G_F}{\sqrt{2}}[V_{ub}V_{uq}^*(c_1O_1^q + c_2O_2^q) - \sum_{i=3}^{10}(V_{ub}V_{uq}^*c_i^u + V_{cb}V_{cq}^*c_i^c + V_{tb}V_{tq}^*c_i^t)O_i^q] + \text{h.c.}\,, \quad (4.6)$$

where the superscript u, c, t indicates the internal quark, and q can be either a d or s quark. The operators O_i^q are defined as

$$\begin{aligned}
O_1^q &= \bar{q}_\alpha\gamma_\mu Lu_\beta\bar{u}_\beta\gamma^\mu Lb_\alpha\,, & O_f^q &= \bar{q}\gamma_\mu Lu\bar{u}\gamma^\mu Lb\,,\\
O_{3,5}^q &= \bar{q}\gamma_\mu Lb\bar{q}'\gamma^\mu L(R)q'\,, & O_{4,6}^q &= \bar{q}_\alpha\gamma_\mu Lb_\beta\bar{q}'_\beta\gamma^\mu L(R)q'_\alpha\,,\\
O_{7,9}^q &= \frac{3}{2}\bar{q}\gamma_\mu Lbe_{q'}\bar{q}'\gamma^\mu R(L)q'\,, & O_{8,10}^q &= \frac{3}{2}\bar{q}_\alpha\gamma_\mu Lb_\beta e_{q'}\bar{q}'_\beta\gamma^\mu R(L)q'_\alpha\,,
\end{aligned} \quad (4.7)$$

where $R(L) = 1 \pm \gamma_5$, and q' is summed over u, d, s. O_2 and O_1 are the tree-level and QCD-corrected operators, respectively. O_{3-6} are the strong gluon-induced penguin operators, and operators O_{7-10} are due to γ and Z exchange (electroweak penguins), and "box" diagrams at loop level. The values of the various Wilson coefficients (WC's) are given by [20]

$$\begin{aligned}
c_1 &= -0.185\,, & c_2 &= 1.082, & & & &\\
c_3^t &= 0.014\,, & c_4^t &= -0.035\,, & c_5^t &= 0.010\,, & &\\
c_6^t &= -0.041\,, & & & & & &\\
c_7^t &= -1.24\times 10^{-5}\,, & c_8^t &= 3.77\times 10^{-4}\,, & c_9^t &= -0.010\,, & c_{10}^t &= 2.06\times 10^{-3}\,.
\end{aligned} \quad (4.8)$$

We will split the contribution to the decay amplitudes in QCD factorization into a factorizable piece and a nonfactorizable piece. This latter part includes the hard spectator interactions and annihilation-type contributions which are formally suppressed by $1/m_b$. We should point out that it is possible that there are additional nonfactorizable contribution to nonleptonic decays, of order Λ_{QCD}/m_b, which are missed in the QCD factorization approach [25]. However the size and methods to calculate such contributions remain a contentious issue and so we will not include them in our estimates of the amplitudes. In any event, the SU(3) breaking from these terms is suppressed by the heavy b-quark mass, and will be small.

We write the general amplitudes $A_{s,d}$ for $\bar{b} \to \bar{s}$ and $\bar{b} \to \bar{d}$ penguin processes as

$$A_{s,d} = A_{s,d}^{\text{fac}} + A_{s,d}^{nonfac}\,, \quad (4.9)$$

where $A_{s,d}^{\rm fac}$ is the factorization contribution and $A_{s,d}^{nonfac}$ is the additional nonfactorizable contribution. In relating A_s and A_d, we write

$$\begin{aligned} A_s^{\rm fac} &= (1+z_{\rm fac})A_d^{\rm fac} , \\ A_s^{nonfac} &= (1+z_{nonfac})A_d^{nonfac} , \end{aligned} \tag{4.10}$$

where $z_{\rm fac}$ and z_{nonfac} are the SU(3)-breaking corrections which are typically $\sim 25\%$. The net SU(3) breaking in the amplitude is then

$$\frac{A_s^{\rm fac}+A_s^{nonfac}}{A_d^{\rm fac}+A_d^{nonfac}} = 1+z_{\rm fac}+(z_{nonfac}-z_{\rm fac})\frac{A_d^{nonfac}}{A_d^{\rm fac}+A_d^{nonfac}} . \tag{4.11}$$

If $z_{nonfac}\sim z_{\rm fac}$, then $z_{\rm fac}$ clearly gives the estimate of SU(3) breaking even in the presence of non-negligible nonfactorizable effects. And if $A_d^{nonfac} \ll A_d^{\rm fac}$, then $z_{\rm fac}$ again gives the estimate of SU(3) breaking. We will use eq. (4.11) to estimate SU(3) breaking from nonfactorizable corrections.

In order to identify the sources of SU(3) breaking, we note that the factorizable contributions to $B_d^0\to\pi^+\pi^-$ and $B_s^0\to K^0\bar{K}^0$ can be written as

$$\begin{aligned} A(B_d^0\to\pi^+\pi^-) &= f_\pi\, F^{B_d^0\to\pi}\int T(x)\phi_\pi(x)dx + 0\left(\frac{1}{m_b}\right) , \\ A(B_s^0\to K^0\bar{K}^0) &= f_K\, F^{B_s^0\to\bar{K}}\int T(x)\phi_K(x)dx + 0\left(\frac{1}{m_b}\right) . \end{aligned} \tag{4.12}$$

In the above, $F^{B_d^0\to\pi}$ and $F^{B_s^0\to\bar{K}}$ are form factors, while the integrals represent the hadronization of quarks into a π or a K^0. The quantities $\phi_{\pi,K}$ are the pion and kaon light cone distributions (LCD's), which can be expanded in terms of Gegenbauer polynomials $C_n^{3/2}$ as follows [26]:

$$\phi_M(x,\mu) = f_K\, 6x(1-x)\left(1+\sum_{n=1}^{\infty}\alpha_n^M(\mu)C_{2n}^{3/2}(2x-1)\right) , \tag{4.13}$$

where the Gegenbauer moments a_{2n}^M are multiplicatively renormalized, change slowly with μ, and vanish as $\mu\to\infty$. The pion LCD is symmetric under $x\to 1-x$ because of isospin symmetry. For the kaon the antisymmetric part of the LCD arises from SU(3) breaking. That is, it is the presence of the antisymmetric piece at scale $\mu\sim m_b$, proportional to odd powers of $(2x-1)$, which will generate SU(3) corrections from the final-state kaons.

In what follows, we denote $B_d^0\to\pi^+\pi^-$ and $B_s^0\to K^0\bar{K}^0$ as pair "a", and $B_d^0\to\pi^+\pi^-$ and $B_u^+\to K^0\pi^+$ as pair "b". Following the notation in ref. [8] the amplitudes for all three decays can be explicitly written as:

$$\begin{aligned} A(B_d^0\to\pi^+\pi^-) &= V_{ub}^*V_{ud}A_{ut}^{\pi\pi} + V_{cb}^*V_{cd}A_{ct}^{\pi\pi} ; \\ A_{ut}^{\pi\pi} &= -\left[a_2+a_{4\pi}^u+a_{10\pi}^u+r_\chi^\pi(a_{6\pi}^u+a_{8\pi}^u)\right]A_{\pi\pi} , \\ A_{ct}^{\pi\pi} &= -\left[a_{4\pi}^c+a_{10\pi}^c+r_\chi^\pi(a_{6\pi}^c+a_{8\pi}^c)\right]A_{\pi\pi} , \\ A_{\pi\pi} &= \frac{iG_F}{\sqrt{2}}(m_B^2-m_\pi^2)F_0^{B_d^0\to\pi}(m_\pi^2)f_\pi . \end{aligned} \tag{4.14}$$

$$
\begin{aligned}
A(B_s^0 \to K^0\bar{K}^0) &= V_{ub}^* V_{us} A_{ut}^{KK} + V_{cb}^* V_{cs} A_{ct}^{KK} \,; \\
A_{ut}^{KK} &= \left[a_{4K}^u - \frac{1}{2} a_{10K}^u + r_\chi^K (a_{6K}^u - \frac{1}{2} a_{8K}^u) \right] A_{KK} \,, \\
A_{ct}^{KK} &= \left[a_{4K}^c - \frac{1}{2} a_{10K}^c + r_\chi^K (a_{6K}^c - \frac{1}{2} a_{8K}^c) \right] A_{KK} \,, \\
A_{KK} &= \frac{iG_F}{\sqrt{2}} (m_{B_s}^2 - m_K^2) F_0^{B_s^0 \to \bar{K}} (m_K^2) f_K \,.
\end{aligned} \tag{4.15}
$$

$$
\begin{aligned}
A(B^+ \to K^0\pi^+) &= V_{ub}^* V_{us} A_{ut}^{K\pi} + V_{cb}^* V_{cs} A_{ct}^{K\pi} \,; \\
A_{ut}^{K\pi} &= \left[a_{4K}^u - \frac{1}{2} a_{10K}^u + r_\chi^K (a_{6K}^u - \frac{1}{2} a_{8K}^u) \right] A_{K\pi} \,, \\
A_{ct}^{K\pi} &= \left[a_{4K}^c - \frac{1}{2} a_{10K}^c + r_\chi^K (a_{6K}^c - \frac{1}{2} a_{8K}^c) \right] A_{K\pi} \,, \\
A_{K\pi} &= \frac{iG_F}{\sqrt{2}} (m_B^2 - m_\pi^2) F_0^{B_d^0 \to \pi} (m_K^2) f_K \,.
\end{aligned} \tag{4.16}
$$

In the above, the a_i are combinations of WC's and are given by $a_i = c_i + c_{i+1}/N_c$ (i odd) or $a_i = c_i + c_{i-1}/N_c$ (i even). The chiral enhancement terms are given by

$$
r_\chi^\pi = \frac{2m_\pi^2}{m_b(m_u + m_d)} \,, \qquad r_\chi^K = \frac{2m_K^2}{m_b(m_u + m_s)} \,. \tag{4.17}
$$

The form factors are defined in eq. (4.4). Note that both $B_d^0 \to \pi^+\pi^-$ and $B^+ \to K^0\pi^-$ involve the same form factor $F_0^{B_d^0 \to \pi}$. This follows from isospin symmetry.

Our methods assume the SU(3) relations

$$
A_{ct}^{\pi\pi} = A_{ct}^{KK} = A_{ct}^{K\pi} \,. \tag{4.18}
$$

From eqs. (4.14), (4.15) and (4.16), we can thus identify three possible sources of SU(3) breaking: (i) π vs. K hadronization. This is represented by differences between the a_{iK} and $a_{i\pi}$, which is related to differences in the π and K LCD's; (ii) the difference in form factors; (iii) differences in the chiral enhancement factors r_χ^π and r_χ^K.

We are now able to compute the SU(3) breaking as

$$
\begin{aligned}
1 + z_{\rm fac}^a &= \frac{A_{ct}^{KK}}{A_{ct}^{\pi\pi}} = \frac{\left[a_{4K}^c - \frac{1}{2} a_{10K}^c + r_\chi^K (a_{6K}^c - \frac{1}{2} a_{8K}^c) \right] A_{KK}}{\left[a_{4\pi}^c + a_{10\pi}^c + r_\chi^\pi (a_{6\pi}^c + a_{8\pi}^c) \right] A_{\pi\pi}} \,, \\
1 + z_{\rm fac}^b &= \frac{A_{ct}^{K\pi}}{A_{ct}^{\pi\pi}} = \frac{\left[a_{4K}^c - \frac{1}{2} a_{10K}^c + r_\chi^K (a_{6K}^c - \frac{1}{2} a_{8K}^c) \right] A_{K\pi}}{\left[a_{4\pi}^c + a_{10\pi}^c + r_\chi^\pi (a_{6\pi}^c + a_{8\pi}^c) \right] A_{\pi\pi}} \,.
\end{aligned} \tag{4.19}
$$

From the values of the WC's in eq. (4.8), we note that the contributions from the color-suppressed electroweak penguins represented by $a_{8,10}$ are tiny. Neglecting these, we can simplify $z_{\rm fac}^{a,b}$ as

$$
\begin{aligned}
1 + z_{\rm fac}^a &= \frac{A_{ct}^{KK}}{A_{ct}^{\pi\pi}} = \frac{\left[a_{4K}^c + r_\chi^K a_{6K}^c \right] A_{KK}}{\left[a_{4\pi}^c + + r_\chi^\pi a_{6\pi}^c \right] A_{\pi\pi}} = \tilde{a}_{K\pi} \frac{A_{KK}}{A_{\pi\pi}} \,, \\
1 + z_{\rm fac}^b &= \frac{A_{ct}^{K\pi}}{A_{ct}^{\pi\pi}} = \frac{\left[a_{4K}^c + r_\chi^K a_{6K}^c \right] A_{K\pi}}{\left[a_{4\pi}^c + r_\chi^\pi a_{6\pi}^c \right] A_{\pi\pi}} = \tilde{a}_{K\pi} \frac{A_{K\pi}}{A_{\pi\pi}} \,.
\end{aligned} \tag{4.20}
$$

In the above, $\tilde{a}_{K\pi}$ represents the SU(3) breaking in the a_i's due to the different LCD's of the kaon and the pion, as well as in r_χ^K/r_χ^π. If the pion and the kaon LCD's take their asymptotic form, then, taking the estimate of $r_\chi^K/r_\chi^\pi = 0.99 \pm 0.06$ from ref. [11], we see that to a very good approximation $\tilde{a}_{K\pi} \approx 1$. The recent measurement of the pion LCD at $\mu^2 \sim 10$ GeV^2 [27] shows that the pion LCD is extremely close to its asymptotic form, $\phi_\pi(x) \sim x(1-x)$. (Note that isospin symmetry requires only that the pion LCD be symmetric, not asymptotic.) This suggests that, at the scale $\mu \sim m_b$, the LCD's of the light mesons K and K^* are probably also very close to their asymptotic form, i.e. symmetric under the interchange $x \leftrightarrow 1-x$. Allowing for SU(3) breaking in the kaon LCD we will take the first Gegenbauer moment to be equal to $\alpha_1^K = 0.2 \pm 0.02$ [11]. The maximum SU(3) breaking then corresponds to $\tilde{a}_{K\pi} = 0.973$, and is only 3%.

One can also consider the vector-vector decays $B_d^0 \to \rho^+\rho^-$, $B_s^0 \to K^{*0}\bar{K}^{*0}$ and $B_u^+ \to K^{*0}\rho^+$. In this case the corresponding value for $\tilde{a}_{K^*\rho}$ is $\tilde{a}_{K^*\rho} = 0.963$, which is only a 4% SU(3) breaking. Note that the SU(3) breaking in the K and K^* LCD's from $\alpha_1^{K(K^*)}$ is very similar: model calculations find the antisymmetric piece to be equal at the level of $\sim 10\%$ [26].

It is clear from the above discussion that the SU(3) breaking in the meson LCD's is small. Henceforth we will neglect these corrections and assume the asymptotic form for the LCD's of the various mesons. We can therefore write

$$1 + z_{\rm fac}^a \approx \frac{A_{KK}}{A_{\pi\pi}} = \frac{(m_{B_s}^2 - m_K^2) F_0^{B_s^0\to\bar{K}}(m_K^2) f_K}{(m_B^2 - m_\pi^2) F_0^{B_d^0\to\pi}(m_\pi^2) f_\pi} \approx \frac{F_0^{B_s^0\to\bar{K}}(m_K^2) f_K}{F_0^{B_d^0\to\pi}(m_\pi^2) f_\pi}, \qquad (4.21)$$

$$1 + z_{\rm fac}^b \approx \frac{A_{K\pi}}{A_{\pi\pi}} = \frac{F_0^{B_d^0\to\pi}(m_K^2) f_K}{F_0^{B_d^0\to\pi}(m_\pi^2) f_\pi} \approx \frac{f_K}{f_\pi}. \qquad (4.22)$$

In the second line, we have assumed that the $B_d^0 \to \pi$ form factor is similar at $q^2 = m_K^2$ and $q^2 = m_\pi^2$. This is reasonable: in $B_d^0 \to \pi$ transitions, where the energy transferred is large, the difference of form factors at $q^2 = m_K^2$ and m_π^2 is suppressed by some power of $m_{K,\pi}^2/m_B^2$ and is therefore negligible.

The factorizable contributions to SU(3) breaking in the decay pairs $B_d^0 \to \pi^+\pi^-$ and $B_s^0 \to K^0\bar{K}^0$, and $B_d^0 \to \pi^+\pi^-$ and $B_u^+ \to K^0\pi^+$, are given by the above expressions. Within QCD factorization, there are (unknown) $1/m_b$ corrections to these results. Their size is typically $(m_s/\Lambda_{QCD}) \times (\Lambda_{QCD}/m_b) = m_s/m_b \sim 5\%$.

We now turn to the nonfactorizable contributions. An example of such an effect is the correction due to hard gluon exchange between the spectator quark and the energetic quarks of the emitted meson. We can parametrize these corrections as

$$A_{\rm non\,fac}^{M_1M_2} = f_B f_{M_1} f_{M_2} \frac{m_B}{\lambda_B} B_{M_1M_2}^H, \qquad (4.23)$$

where $m_{B_q}/\lambda_{B_q} = \int \phi_{B_q}(z)/z$, in which $\phi_{B_q}(z)$ is the B_q^0 LCD, $q = d, s$. The quantity $B_{M_1M_2}^H$ depends on the final state and cannot be calculated as it suffers from endpoint divergences that will eventually be smoothed out by unknown soft physics. In the approach of ref. [8], the divergent piece is regulated by an unknown parameter which is then assumed to be within a certain range. Here we adopt this same method.

We can now estimate the size of nonfactorizable SU(3) breaking in the pairs of processes "a" and "b":

$$
\begin{aligned}
1+z^a_{\text{non fac}} &= \frac{A^{KK}_{ct}}{A^{\pi\pi}_{ct}} = \frac{f_{B_s}}{f_{B_d}}\frac{\lambda_{B_d}}{\lambda_{B_s}}\frac{f_K^2}{f_\pi^2}\frac{m_{B_s}}{m_B}\frac{B^H_{KK}}{B^H_{\pi\pi}}\,,\\
1+z^b_{\text{non fac}} &= \frac{A^{K\pi}_{ct}}{A^{\pi\pi}_{ct}} = \frac{f_K}{f_\pi}\frac{B^H_{K\pi}}{B^H_{\pi\pi}}\,.
\end{aligned}
\tag{4.24}
$$

There are two sources of SU(3) breaking. The first comes from the ratios of the $B^H_{M_1M_2}$. We assume that both $B^H_{KK}/B^H_{\pi\pi}$ and $B^H_{K\pi}/B^H_{\pi\pi}$ are $\sim 1\pm 0.25$. (The error of 25% (typical of SU(3) breaking) is somewhat larger than the estimate using the approach in ref. [8].) The second comes from the initial state. This SU(3) breaking, z_i, is given by

$$
z_i = \frac{f_{B_s}}{f_{B_d}}\frac{\lambda_{B_d}}{\lambda_{B_s}}\,. \tag{4.25}
$$

In a simple model one can write $f_{B_q} = \mu_q^{3/2}/m_{B_q}^{1/2}$ and $\lambda_{B_q}\sim\mu_q$, where μ_q is the reduced mass, which is different for the B^0_s and the B^0_d mesons. In the heavy-quark limit we have $f_{B_s}/f_{B_d} = \mu_s^{3/2}/\mu_d^{3/2}$ and $(f_{B_s}/\lambda_{B_s})/(f_{B_d}/\lambda_{B_d}) = \mu_s^{1/2}/\mu_d^{1/2}$. Taking $f_{B_s}/f_{B_d} = 1.15$, we find that the initial-state SU(3)-breaking correction $z_i - 1$ is $\sim 5\%$.

Using eqs. (4.11), (4.21) and (4.22), we can then write the total SU(3)-breaking correction as

$$
\begin{aligned}
z^a_{\text{total}} &= \frac{A^{KK}_{ct}}{A^{\pi\pi}_{ct}} - 1 \approx z^a_{\text{fac}} - (1+z^a_{\text{fac}})\left[1 - z_i\frac{f_K}{f_\pi}\frac{F_0^{B^0_d\to\pi}(m_\pi^2)}{F_0^{B^0_s\to\bar K}(m_K^2)}\frac{B^H_{KK}}{B^H_{\pi\pi}}\right]\frac{A^{\pi\pi}_{\text{non fac}}}{A^{\pi\pi}_{fac}+A^{\pi\pi}_{\text{non fac}}}\\
z^b_{\text{total}} &= \frac{A^{K\pi}_{ct}}{A^{\pi\pi}_{ct}} - 1 \approx z^b_{\text{fac}} - (1+z^b_{\text{fac}})\left[1 - \frac{B^H_{K\pi}}{B^H_{\pi\pi}}\right]\frac{A^{\pi\pi}_{\text{non fac}}}{A^{\pi\pi}_{fac}+A^{\pi\pi}_{non\,fac}}\,.
\end{aligned}
\tag{4.26}
$$

The calculation of ref. [8] indicates that the hard spectator correction is small, typically about 10% or less in the decays we are considering. Combined with the fact that we expect $(f_K/f_\pi)(F_0^{B^0_d\to\pi}(m_\pi^2)/F_0^{B^0_s\to\bar K}(m_K^2)) \sim 1$ in the second term in z^a_{total} above, we find that the SU(3) corrections from the hard spectator corrections are $\sim 3\%$, which is quite small.

We therefore conclude that, to the extent that annihilation-type topologies are unimportant, the principal SU(3)-breaking effect in $B^0_d\to\pi^+\pi^-$ and $B^0_s\to K^0\bar K^0$ or $B^0_d\to\pi^+\pi^-$ and $B^+_u\to K^0\pi^+$ comes from the factorizable contributions to the decays (eqs. (4.21) and (4.22)).

4.3 $B^0_d\to K^0\bar K^0$

There are two candidate partner processes for $B^0_d\to K^0\bar K^0$: $B^0_s\to K^0\bar K^0$ and $B^+_u\to K^0\pi^+$. Below we analyze separately the SU(3) breaking in each of these pairs of decays.

We consider first $B^0_d\to K^0\bar K^0$ and $B^0_s\to K^0\bar K^0$ [4]. Following the analysis of the previous section we estimate the SU(3) breaking in this pair of decays to be

$$
1+z_{\text{fac}} \approx \frac{A^s_{KK}}{A^d_{KK}} = \frac{F_0^{B^0_s\to\bar K}(m_K^2)}{F_0^{B^0_d\to K}(m_K^2)}\,. \tag{4.27}
$$

Since the final states are the same for both decays, there is no SU(3) breaking due to decay constants.

In the case of second pair of decay processes, $B_d^0 \to K^0\bar{K}^0$ and $B_u^+ \to K^0\pi^+$, the analysis of the previous subsection gives

$$1 + z_{\rm fac} \approx \frac{A_{K\pi}}{A_{KK}} = \frac{F_0^{B_d^0\to\pi}(m_K^2)}{F_0^{B_d^0\to K}(m_K^2)} \,. \tag{4.28}$$

4.4 $B_d^0 \to \rho^0\rho^0$

The partner process to $B_d^0 \to \rho^0\rho^0$ is $B_d^0 \to K^{*0}\rho^0$. Since in this case we are dealing with vector-vector final states, we have to consider specific helicity states. In the linear polarization basis there are three independent polarization amplitudes. They are A_0 (longitudinal amplitude) and $A_{\|,\perp}$ (two transverse amplitudes). Ignoring small differences in the WC's for the two processes, the SU(3)-breaking term for a given polarization state $\lambda = 0, \perp, \|$ is given by

$$1 + z_{\rm fac}^{\lambda} = \frac{\left[a_4^c - \frac{3}{2}a_9^c - \frac{1}{2}a_{10}^c\right] A_{\rho\rho}^{\lambda}}{\left[a_4^c - \frac{1}{2}a_{10}^c\right] A_{K^*\rho}^{\lambda} + \left[-\frac{3}{2}a_9^c\right] A_{\rho K^*}^{\lambda}} \,. \tag{4.29}$$

Within QCD factorization, one must now express the polarization amplitudes $A_{\rho\rho}^{\lambda}$, $A_{K^*\rho}^{\lambda}$ and $A_{\rho K^*}^{\lambda}$ in terms of masses, decay constants and form factors. We do this below.

The polarization amplitudes $A_{V_1V_2}^{\lambda}$ for the process $B \to V_1V_2$ are given as [28]:

$$\begin{aligned}
A_{V_1V_2}^{\|} &= \sqrt{2}\, a_{V_1V_2} \\
A_{V_1V_2}^{0} &= -a_{V_1V_2}x - \frac{m_1m_2}{m_B^2}\, b_{V_1V_2}(x^2-1)\,, \\
A_{V_1V_2}^{\perp} &= 2\sqrt{2}\,\frac{m_1m_2}{m_B^2} c_{V_1V_2}\sqrt{x^2-1}\,,
\end{aligned} \tag{4.30}$$

with $x = k_1 \cdot k_2/(m_1m_2)$, where $k_{1,2}$ and $m_{1,2}$ are the momenta and the masses of the vector mesons $V_{1,2}$. The parameters $a_{V_1V_2}$, $b_{V_1V_2}$ and $c_{V_1V_2}$ can be written [28]

$$\begin{aligned}
a_{V_1V_2} &= -m_1 g_{V_1}(m_B+m_2)A_1^{(2)}(m_1^2)\,, \\
b_{V1V_2} &= 2m_1 g_{V_1}\frac{m_B}{(m_B+m_2)} m_B A_2^{(2)}(m_1^2)\,, \\
c_{V_1V_2} &= -m_1 g_{V_1}\frac{m_B}{(m_B+m_2)} m_B V^{(2)}(m_1^2)\,.
\end{aligned} \tag{4.31}$$

These quantities depend on the form factors $V^{(2)}$ and $A_{1,2}^{(2)}$, and on the decay constant of V_1, g_{V_1}. These are defined as follows.

A $B \to V_i$ transition is described by the four form factors $V^{(i)}$, $A_{0,1,2}^{(i)}$ [21]:

$$\begin{aligned}
\langle V_i(k_i,\lambda)|\, \bar{q}'\gamma_\mu b\,|B(p)\rangle &= i\frac{2V^{(i)}(q^2)}{(m_B+m_i)}\epsilon_{\mu\nu\rho\sigma}p^\nu k_i^\rho \varepsilon_\lambda^{*\sigma}\,, \\
\langle V_i(k_i,\lambda)|\, \bar{q}'\gamma_\mu\gamma_5 b\,|B(p)\rangle &= (m_B+m_i)A_1^{(i)}(q^2)\left[\varepsilon_{\mu,\lambda}^* - \frac{\varepsilon_\lambda^*{\cdot}q}{q^2}q_\mu\right] - \\
&\quad -A_2^{(i)}(q^2)\frac{\varepsilon_\lambda^*{\cdot}q}{m_B+m}\left[(p_\mu+k_\mu^\lambda) - \frac{m_B^2-m_i^2}{q^2}q_\mu\right] + \\
&\quad +2m_i\frac{\varepsilon_\lambda^*{\cdot}q}{q^2}q_\mu A_0^{(i)}(q^2)\,,
\end{aligned} \tag{4.32}$$

where $q = p - k_i$. Although the values of these form factors are model-dependent, the number of independent form factors can be reduced if the dominant contribution to the form factors comes from soft gluon interactions between the quarks inside the mesons. In this case, in the limit $m_b \to \infty$ and $E_V \to \infty$, one has the following relations between the vector form factors [29]:

$$
\begin{aligned}
A_1(q^2) &= \frac{2E_V}{m_B + m_V}\zeta_\perp(m_B, E_V)\,, \\
A_2(q^2) &= (1+\frac{m_V}{m_B})\zeta_\perp(m_B, E_V)\left[1 - \frac{m_V}{E_V}\frac{\zeta_{||}(m_B, E_V)}{\zeta_\perp(m_B, E_V)}\right], \\
V(q^2) &= (1+\frac{m_V}{m_B})\zeta_\perp(m_B, E_V)\,.
\end{aligned}
\tag{4.33}
$$

From this we see that the form factors $A_{1,2}$ and V are expressible in terms of two form factors $\zeta_{\perp,||}$. Moreover, we see that the difference between $\zeta_{||}$ and $\zeta_\perp$ is suppressed by m_V/E_V. Coupled with the fact that model calculations indicate that $\zeta_{||} \sim \zeta_\perp$, we can assume that $\zeta_{||} = \zeta_\perp$ for the purpose of calculating SU(3) breaking.

The decay constant of a vector mesons is defined by $m_V g_V \epsilon^*_\mu \sim \langle V(p,\epsilon)|\, J_\mu \,|0\rangle$. However, the size of the polarization vector ϵ^*_μ differs for different polarizations: for transverse polarization $\epsilon \sim 0(1)$, while for longitudinal polarization $\epsilon \sim E_V/m_V$. This must be taken into account when evaluating the $A_{V_1V_2}$.

Putting all of the above information together, we can obtain expressions for $A^\lambda_{\rho\rho}$, $A^\lambda_{K^*\rho}$ and $A^\lambda_{\rho K^*}$. Note that $A^\lambda_{K^*\rho} \sim g_{K^*}F^{B^0_d\to\rho}$ and $A^\lambda_{\rho K^*} \sim g_\rho F^{B^0_d\to K^*}$, where $F^{B^0_d\to\rho}$ and $F^{B^0_d\to K^*}$ are form factors. Since $F^{B^0_d\to K^*}/F^{B^0_d\to\rho} \sim g_{K^*}/g_\rho$, we have $A^\lambda_{K^*\rho} \simeq A^\lambda_{\rho K^*}$. Combined with the fact that the EWP WC a^c_9 is smaller than the QCD penguin WC a^c_4, we can simplify eq. 4.29 as

$$
1 + z^\lambda_{\rm fac} = \frac{\left[a^c_4 - \frac{3}{2}a^c_9 - \frac{1}{2}a^c_{10}\right]A^\lambda_{\rho\rho}}{\left[a^c_4 - \frac{3}{2}a^c_9 - \frac{1}{2}a^c_{10}\right]A^\lambda_{K^*\rho} + \frac{3}{2}a^c_9\left[A^\lambda_{\rho K^*} - A^\lambda_{\rho K^*}\right]} \approx \frac{A^\lambda_{\rho\rho}}{A^\lambda_{K^*\rho}}\,. \tag{4.34}
$$

Both $A^\lambda_{\rho\rho}$ and $A^\lambda_{K^*\rho}$ involve the same $B \to \rho$ form factors, so that here SU(3) breaking depends only on the ratios of decay constants and masses of the K^* and ρ. This can be seen easily as follows: neglecting terms of $O(m^2_{\rho,K^*}/m^2_B)$, the expressions for the polarization amplitudes simplify [28]:

$$
\begin{aligned}
A^{||}_{V_1V_2} &= \sqrt{2}\,a_{V_1V_2} \\
A^0_{V_1V_2} &= -(2a_{V_1V_2} + b_{V_1V_2})\frac{m^2_B}{4m_1m_2}\,, \\
A^\perp_{V_1V_2} &= \sqrt{2}\,c_{V_1V_2}\,.
\end{aligned}
\tag{4.35}
$$

The SU(3)-breaking effects for the different polarization states are then given as

$$
1 + z^0_{\rm fac} = \frac{g_\rho}{g_{K^*}}\,; \qquad 1 + z^{\perp,||}_{\rm fac} = \frac{m_\rho g_\rho}{m_{K^*}g_{K^*}}\,. \tag{4.36}
$$

4.5 $B_s^0 \to \bar{K}^{*0}\rho^0$

Here there is a single candidate partner process: $B_d^0 \to K^{*0}\rho^0$. The SU(3) breaking is given by

$$1 + z_{\rm fac}^{\lambda} = \frac{\left[a_4^c - \frac{3}{2}a_9^c - \frac{1}{2}a_{10}^c\right] A_{\rho K^*}^{s,\lambda}}{\left[a_4^c - \frac{1}{2}a_{10}^c\right] A_{K^*\rho}^{\lambda} + \left[-\frac{3}{2}a_9^c\right] A_{\rho K^*}^{\lambda}}\,, \tag{4.37}$$

where $A_{K^*\rho}^{\lambda}$ and $A_{\rho K^*}^{\lambda}$ are given in eq. (4.30), and $A_{\rho K^*}^{s,\lambda}$ is obtained from $A_{\rho K^*}^{\lambda}$ by replacing B_d^0 by B_s^0. As before we can approximate the SU(3) breaking as

$$1 + z_{\rm fac}^{\lambda} \approx \frac{A_{\rho K^*}^{s,\lambda}}{A_{K^*\rho}^{\lambda}}\,. \tag{4.38}$$

Using the expressions given in the previous subsection, this yields

$$\begin{aligned}
1 + z_{\rm fac}^{0} &= \frac{m_{B_s^0}^2}{m_{B_d^0}^2}\frac{m_\rho\, g_\rho}{m_{K^*}\, g_{K^*}}\frac{(m_{B_d^0} + m_\rho)}{(m_{B_s^0} + m_{K^*})}\times \\
&\quad\times\frac{-(m_{B_s^0} + m_{K^*})^2\, A_1^{B_s^0\to K^*}(m_\rho^2) + m_{B_s^0}^2\, A_2^{B_s^0\to K^*}(m_\rho^2)}{-(m_{B_d^0} + m_\rho)^2\, A_1^{B_d^0\to\rho}(m_{K^*}^2) + m_{B_d^0}^2\, A_2^{B_d^0\to\rho}(m_{K^*}^2)}\,, \\
1 + z_{\rm fac}^{\|} &= \frac{m_\rho\, g_\rho\,(m_{B_s^0} + m_{K^*})\, A_1^{B_s^0\to K^*}(m_\rho^2)}{m_{K^*}\, g_{K^*}\,(m_{B_d^0} + m_\rho)\, A_1^{B_d^0\to\rho}(m_{K^*}^2)}\,, \\
1 + z_{\rm fac}^{\perp} &= \frac{m_\rho\, g_\rho\, m_{B_s^0}^2\,(m_{B_d^0} + m_\rho)\, V^{B_s^0\to K^*}(m_\rho^2)}{m_{K^*}\, g_{K^*}\,(m_{B_s^0} + m_{K^*})\, V^{B_d^0\to\rho}(m_{K^*}^2)}\,.
\end{aligned} \tag{4.39}$$

We see that here the SU(3)-breaking term depends on form factors, as well as decay constants and vector-meson masses.

4.6 $B_s^0 \to \phi\bar{K}^{*0}$

In this case there are three candidate partner processes: $B_d^0 \to \phi K^{*0}$, $B_u^+ \to \phi K^{*+}$ and $B_s^0 \to \phi\phi$. We refer to the three pairs of decays, in this order, as pair "a", "b" and "c". The SU(3)-breaking term for both pairs "a", and "b" is given by

$$1 + z_{\rm fac}^{\lambda} = \frac{\left[a_3^c + a_4^c + a_5^c - \frac{1}{2}a_7^c - \frac{1}{2}a_9^c - \frac{1}{2}a_{10}^c\right] A_{\phi K^*}^{\lambda}}{\left[a_4^c - \frac{1}{2}a_{10}^c\right] A_{\bar{K}^*\phi}^{s,\lambda} + \left[a_3^c + a_5^c - \frac{1}{2}a_7^c - \frac{1}{2}a_9^c\right] A_{\phi\bar{K}^*}^{s,\lambda}}\,. \tag{4.40}$$

The SU(3) breaking for "c" has the same form as the above, but with the $A_{\phi K^*}^{\lambda}$ in the numerator being replaced by $A_{\phi\phi}^{s,\lambda}$. Previous arguments tell us that $A_{\bar{K}^*\phi}^{s,\lambda}$ and $A_{\phi\bar{K}^*}^{s,\lambda}$ are of similar size. Thus, using the fact that the WC's $a_{3,5}^c$ and $a_{7,9}^c$ are much smaller than the QCD penguin WC a_4^c, we can rewrite eq. 4.40 as

$$1 + z_{\rm fac}^{a,\lambda} \approx 1 + z_{\rm fac}^{b,\lambda} \approx \frac{A_{\phi K^*}^{\lambda}}{A_{\bar{K}^*\phi}^{s,\lambda}}\,; \qquad 1 + z_{\rm fac}^{c,\lambda} \approx \frac{A_{\phi\phi}^{s,\lambda}}{A_{\bar{K}^*\phi}^{s,\lambda}}\,. \tag{4.41}$$

For pairs "a" and "b", the SU(3)-breaking term includes both decay constants and form factors, as in eq. (4.39). However, for pair "c", all dependence on form factors vanishes, and we have

$$1 + z_{\rm fac}^{c,0} = \frac{g_\phi}{g_{K^*}}\,; \qquad 1 + z_{\rm fac}^{c,\perp,\parallel} = \frac{m_\phi g_\phi}{m_{K^*} g_{K^*}}\,. \tag{4.42}$$

4.7 Annilation-type contributions

The amplitudes for most pairs of decays discussed previously are equal in the SU(3) limit only if annihilation-type amplitudes are neglected. Thus, for these decays, it is unnecessary to consider SU(3) breaking in such contributions. Note that, as mentioned earlier, this assumption can be tested experimentally. If it turns out that such amplitudes are large, perhaps because of chiral enhancements, then one can use instead vector-vector final states for which annihilation-type contributions are expected to be small.

There are, however, three pairs of decays whose amplitudes are equal in the SU(3) limit: $B_d^0 \to D^+ D^-$ and $B_s^0 \to D_s^+ D_s^-$, $B_d^0 \to K^0 \bar{K}^0$ and $B_s^0 \to K^0 \bar{K}^0$, and $B_s^0 \to \phi \bar{K}^{*0}$ and $B_d^0 \to \phi K^{*0}$. For these decays, one need not neglect annihilation-type amplitudes. In this case, one can consider the size of SU(3)-breaking effects in such contributions. Below, we present an estimate of this SU(3) breaking within QCD factorization for $B_d^0 \to K^0 \bar{K}^0$ and $B_s^0 \to K^0 \bar{K}^0$.

The annihilation terms for these decays can be parametrized as

$$\begin{aligned} A_{\rm ann}^{s,\bar{K}K} &= f_{B_s} f_K f_K X_{KK}^s\,, \\ A_{\rm ann}^{d,\bar{K}K} &= f_{B_d} f_K f_K X_{KK}^d\,. \end{aligned} \tag{4.43}$$

Like the hard spectator corrections, the quantities $X_{KK}^{s,d}$ depend on the kaon LCD and are divergent because of missing soft contributions. Within the QCD factorization approach

$$X_{KK}^{s,d} \sim r_\chi^K (2X_{s,d}^2 - X_{s,d})\,, \tag{4.44}$$

where the divergent quantities $X_{s,d}$ are regulated as

$$X_{s,d} = (1 + \rho_{s,d} e^{i\phi_{s,d}})\, \ln \frac{m_B}{\Lambda_{QCD}}\,. \tag{4.45}$$

ρ_s and ρ_d are related by SU(3), as are ϕ_s and ϕ_d. However, these quantities are not known; they are taken to satisfy $|\rho_{s,d}| < 1$ [8]. The SU(3) breaking in eq. (4.43) can then be estimated by assuming that $\rho_{d,s}$ and $\phi_{d,s}$ differ by 25%.

Thus, including annihilation terms, the size of the SU(3) corrections can be estimated to be

$$z_{\rm total} = \frac{A_{s,ct}^{KK}}{A_{d,ct}^{KK}} - 1 \approx z_{\rm fac} - (1 + z_{\rm fac}) \left[1 - \frac{f_{B_s}}{f_B} \frac{F_0^{B_d^0 \to K^0}(m_K^2)}{F_0^{B_s^0 \to \bar{K}}(m_K^2)} \frac{X_{KK}^s}{X_{KK}^d} \right] R_A\,, \tag{4.46}$$

where

$$R_A = \frac{A_{\rm ann}^{d,KK}}{A_{fac}^{d,KK} + A_{\rm ann}^{d,KK}}\,. \tag{4.47}$$

Process	Partner Process	Amp. Error	SU(3) Breaking
$B_d^0 \to D^+D^-$	$B_s^0 \to D_s^+D_s^-$	0	$\frac{f_{D_s}}{f_D}\frac{F_0^{B_s^0\to D_s}}{F_0^{B_d^0\to D}}$
	$B_d^0 \to D_s^+D^-$	$\sim 5\%$	$\frac{f_{D_s}}{f_D}$
	$B_u^+ \to D_s^+\bar{D}^0$	$\sim 5\%$	$\frac{f_{D_s}}{f_D}$
$B_d^0 \to \pi^+\pi^-$	$B_s^0 \to K^0\bar{K}^0$	$\sim 5\%$	$\frac{f_K}{f_\pi}\frac{F_0^{B_s^0\to K}}{F_0^{B_d^0\to \pi}}$
	$B_u^+ \to K^0\pi^+$	$\sim 10\%$	$\frac{f_K}{f_\pi}$
$B_d^0 \to K^0\bar{K}^0$	$B_s^0 \to K^0\bar{K}^0$	0	$\frac{F_0^{B_s^0\to \bar{K}}}{F_0^{B_d^0\to K}}$
	$B_u^+ \to K^0\pi^+$	$\sim 5\%$	$\frac{F_0^{B_d^0\to \pi}}{F_0^{B_d^0\to K}}$
$B_d^0 \to \rho^0\rho^0$	$B_d^0 \to K^{*0}\rho^0$	$\sim 10\%$	$\frac{g_\rho}{g_{K^*}}$
$B_s^0 \to \bar{K}^{*0}\rho^0$	$B_d^0 \to K^{*0}\rho^0$	$\sim 5\%$	$\frac{g_\rho}{g_{K^*}}\frac{F^{B_s^0\to \bar{K}^*}}{F^{B_d^0\to \rho}}$
$B_s^0 \to \phi\bar{K}^{*0}$	$B_d^0 \to \phi K^{*0}$	0	$\frac{g_\phi}{g_{K^*}}\frac{F^{B_d^0\to K^*}}{F^{B_s^0\to \phi}}$
	$B_u^+ \to \phi K^{*+}$	$\sim 1\%$	$\frac{g_\phi}{g_{K^*}}\frac{F^{B_d^0\to K^*}}{F^{B_s^0\to \phi}}$
	$B_s^0 \to \phi\phi$	$\sim 5\%$	g_ϕ/g_{K^*}

Table 1: B decays and their partner processes which can be used to obtain CP phase information with Method I or II. The theoretical uncertainties due to the neglect of certain amplitudes (Amp. Error) and SU(3) breaking are shown. For most decays, the expressions for the SU(3)-breaking error also include (known) mass factors. For the vector-vector final states, the "form factor" $F^{B\to f}$ is symbolic only: see eqs. (4.39) and (4.41) for the exact expressions.

The total amount of SU(3) breaking clearly depends on the size of the annihilation contributions. If the annihilation terms are large — ref. [18] estimates them to be $\sim 40\%$ of the penguin amplitude — then the SU(3) breaking due to such terms could be 10-15% in $\bar{b} \to \bar{s}$ decays.

5. Discussion

Our findings are summarized in table 1. There are six neutral $B^0 \to M_1M_2$ decays involving a $\bar{b} \to \bar{d}$ penguin amplitude in which both B^0 and $\overline{B}^0$ mesons can decay to M_1M_2. For each of these decays, we list the potential partner processes $B' \to M_1'M_2'$ which receive a significant $\bar{b} \to \bar{s}$ penguin contribution and which are dominated by a single amplitude. There are a total of twelve decay pairs to which our methods can be applied.

For each pair, there are two main sources of theoretical uncertainty. First, there is the "amplitude error," which corresponds to the error due to the neglect of certain diagrams, usually of exchange/annihilation type. These diagrams have been estimated to be at most

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$O(\bar{\lambda}^2) \sim 5\%$ of the leading decay amplitude (eqs. (3.4) and (3.5)). In most cases one diagram must be neglected, leading to an amplitude error of $\sim 5\%$. However, there are some decay pairs for which this error is zero (no diagrams neglected) or $\sim 10\%$ (two diagrams neglected).

As noted earlier, in some approaches to hadronic B decays [8, 18], annihilation-type diagrams may be enhanced for final states involving pseudoscalars due to "chiral enhancement" factors such as those found in eq. (4.17). If true — and this can be tested experimentally — then the amplitude errors given in table 1 may be underestimated for certain decays. In this case, one can avoid chiral enhancements by considering only decays with vector-vector final states. For such decays, an angular analysis will be necessary to separate out the three different helicity states. Method I can then be applied to a single such state, while two helicity states are required for Method II.

The second source of theoretical uncertainty is the breaking of flavor SU(3) symmetry in the ratio of eq. (2.7), which is expected to be $O(m_s/\Lambda_{QCD}) \sim 25\%$. In order for our methods to be useful, this SU(3) uncertainty must be reduced. We have estimated the SU(3) breaking for each pair of decays, and it appears in the last column of table 1. For five decay pairs, the SU(3) breaking is given solely by a ratio of decay constants, sometimes multiplied by a function of (known) masses. Many of these decay constants have been measured experimentally. For pseudoscalars, we have $f_\pi = 131\,\mathrm{MeV}$ and $f_K = 160\,\mathrm{MeV}$ [1]; for vectors, $g_\rho = 209\,\mathrm{MeV}$, $g_{K^*} = 218\,\mathrm{MeV}$ and $g_\phi = 221\,\mathrm{MeV}$ [30]. The ratios f_K/f_π, g_ρ/g_{K^*} and g_ϕ/g_{K^*} are therefore all known, with small errors. There are also experimental values for another ratio, f_{D_s}/f_D, but the errors are huge [1]. Instead, one can take the value of this ratio from the lattice: $f_{D_s}/f_D = 1.22 \pm 0.04$ [23], which has a very small error. Thus, for the five decay pairs in which the leading SU(3)-breaking term is given by a ratio of decay constants, the unknown theoretical SU(3) uncertainty in eq. (2.7) is only a second-order effect. In this case, one can use Method I to extract CKM phase information from these pairs of decays. From an experimental point of view, these are the favored pairs of decay modes. In section 6, using the latest data, we show explictly how Method I can be applied to $B_d^0 \to \pi^+\pi^-$ and $B_u^+ \to K^0\pi^+$ decays to extract γ.

For the remaining seven decay pairs, the SU(3) breaking is given by an expression involving form factors. Unfortunately, these form factors cannot be calculated yet in QCD, and so one must find a way of estimating them in order to reduce the size of SU(3) breaking. One way is to simply rely on model calculations. However, it is difficult to argue that the theoretical error is smaller than the canonical size of SU(3) breaking, $\sim 25\%$.

In some cases, the ratio of form factors can be *measured* using the partner processes. For example, the ratio of amplitudes for $B_d^0 \to D_s^+ D^-$ and $B_s^0 \to D_s^+ D_s^-$ is proportional to $F_0^{B_d^0 \to D}/F_0^{B_s^0 \to D_s}$. Thus, to a good approximation, the measurement of the rates for these processes allows us to measure the desired ratio of form factors. Similarly, $F_0^{B_s^0 \to \bar{K}}/F_0^{B_d^0 \to \pi}$ and $F^{B_d^0 \to \bar{K}^*}/F^{B_s^0 \to \phi}$ can be obtained from measurements of the partner processes $B_s^0 \to K^0\bar{K}^0$ and $B_u^+ \to K^0\pi^+$, and $B_d^0 \to \phi K^{*0}$ and $B_s^0 \to \phi\phi$, respectively. One can therefore extract ratios of form factors from the measurement of the branching ratios for two $\bar{b} \to \bar{s}$ penguin decays, up to the theoretical error incurred by neglecting annihilation-type diagrams.

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Another way to measure the ratio of form factors is to consider the heavy-quark limit $m_{b,c} \to \infty$. In this case, the ratio of form factors in B decays is related to a similar ratio in D decays. That is,

$$\frac{F^{B_s^0 \to f_1}}{F_{B_d^0 \to f_2}} = \frac{F^{\bar{D}_s \to f_1}}{F_{\bar{D} \to f_2}}, \qquad \frac{F^{B_d^0 \to f_1}}{F_{B_d^0 \to f_2}} = \frac{F^{\bar{D} \to f_1}}{F_{\bar{D} \to f_2}}. \tag{5.1}$$

The D form-factor ratios can be measured in semileptonic D decays. The correction to the relation between B and D form factors is $O(1/m_{b,c})$. Most of the form-factor ratios in table 1 can in principle be measured in this way.

If it is not possible to get information about the form-factor ratios using the above methods, one can reduce the error due to SU(3) breaking by using Method II. In this case one uses two decay pairs related by SU(3). The double ratio of decay amplitudes then provides the necessary input to extract CP-phase information.

Several decay pairs in table 1 involve pseudoscalar final states. In applying Method II to these decays, it is tempting to take as the second decay pair the corresponding vector-vector decay. However, for $B \to$ *light* transition, this will not work well: there are no relations between $B \to P$ and and $B \to V$ form factors. Thus, though intuitively we expect some cancellation when we consider the double ratio of form factors, the amount of cancellation is model-dependent.

There are two exceptions to this. First, in $B \to$ *heavy* transitions all the $B \to P$ and $B \to V$ form factors can be expressed in terms of a single Isgur-Wise function in the $m_{b,c} \to \infty$ limit. Thus, for the decays $B_d^0 \to D^{(*+)} D^{(*-)}$ and $B_s^0 \to D_s^{(*+)} D_s^{(*-)}$, we can use Method II with pseudoscalar and vector final states, and the SU(3) breaking will cancel in the $m_{b,c} \to \infty$ limit. The SU(3)-breaking effects from finite $m_{b,c}$ will be suppressed by $m_s/m_{b,c}$.

The second exception is the decay pair $B_d^0 \to K^0 \bar{K}^0$ and $B_s^0 \to K^0 \bar{K}^0$. This case is special because the final state is the same for both processes. Here the ratio of pseudoscalar form factors differs from unity (for a symmetric kaon LCD) only due to the initial-state correction which arises because of the difference in the light degrees of freedom in the B_d^0 and B_s^0 mesons. A similar correction will appear in the ratio of vector form factors. Indeed, using the QCD sum rule model of form factors, the initial-state correction is the same for both P and V form factors, so that it cancels in the double ratio. We therefore find that, to leading order, there is no SU(3) breaking in Method II [4].

Apart from these two cases, Method II works best when two polarization states of the VV final state are used. For the remaining ($B \to$ *light*) decays, to the extent that eq. (4.33) is valid, the three polarization amplitudes in the principal $B \to VV$ decay, as well as its partner process, are expressible in terms of a single universal form factor. Thus, all dependence on form factors vanishes in the double ratio, leaving only a theoretical uncertainty at the level of second-order SU(3) breaking. (Note: we have found that a $\pm 30\%$ difference in $\zeta_{\|}$ and $\zeta_{\perp}$ in eq. (4.33) does not significantly affect the calculation of SU(3) breaking.)

However, several caveats must be pointed out. First, it is not certain that the assumptions behind the relations in eq. (4.33) are valid. It is necessary to measure the form factors in semileptonic B decays to test the validity of eq. (4.33). If these relations are not found

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to be valid, then, while we expect that much of the SU(3) breaking in the double ratio will cancel, this is not guaranteed by any symmetry principle. (However, such cancellations occur to varying degrees in most form-factor models.)

It is also possible to use Method II for decay pairs in which the main SU(3)-breaking uncertainty is due to decay constants. However, this is unnecessary, since the values of the decay constants are known.

It is clear that Method II will only apply to decays which have more than one polarization state. Thus, it will not apply to the decay $B \to \rho^+\rho^-$, which has been found to be dominated by the longitudinal polarization [31]. On the other hand, the measurement of the polarization amplitudes in $B_d^0 \to \phi K^*$ indicates a sizeable transverse component [31], so that Method II should be feasible in this case.

Finally, we address the question of which of the twelve B decay pairs is most promising for extracting CKM phase information. As noted earlier, for five pairs the leading theoretical uncertainty is given by a ratio of decay constants only, and is therefore already known. Of these, one pair — $B_s^0 \to \phi\bar{K}^{*0}$ and $B_s^0 \to \phi\phi$ — involves B_s^0 decays. The measurements of these decays will eventually be made by future hadron colliders, but this will take a number of years. A second pair — $B_d^0 \to \rho^0\rho^0$ and $B_d^0 \to K^{*0}\rho^0$ — requires an angular analysis. In addition, the branching ratios for these decays are expected to be small, so these measurements will also take some time. The most promising applications of our methods therefore involve the remaining three decays. For one of these — $B_d^0 \to \pi^+\pi^-$ and $B_u^+ \to K^0\pi^+$ — data is already available. We present the analysis of this pair of decays in section 6. Measurements of CP violation in the other decay pairs — $B_d^0 \to D^+D^-$ and $B_d^0 \to D_s^+D^-$ [5] or $B_u^+ \to D_s^+\bar{D}^0$ — will likely be made soon.

To conclude, we have found a number of pairs of B decays, related by flavor SU(3), whose measurements can be combined to furnish information about CP phases. The main source of theoretical uncertainty in relating the decays $B^0 \to M_1M_2$ and $B' \to M_1'M_2'$ is SU(3) breaking, which is typically $O(m_s/\Lambda_{QCD}) \sim 25\%$. We have given several ways of removing the leading-order SU(3) uncertainty. This leaves a second-order error of $\sim$ 5%. Depending on the decay pair chosen, and how one combines the various theoretical uncertainties, the theoretical error is between 5% and 15%.

6. Extracting γ from $B_d^0 \to \pi^+\pi^-$ and $B_u^+ \to K^0\pi^+$

In this section, using the latest data, we show explictly how our Method I can be applied to $B_d^0 \to \pi^+\pi^-$ and $B_u^+ \to K^0\pi^+$ decays to obtain the CP phase γ. (This is basically an update of ref. [6].) As we will see, although the method is in principle feasible, the present experimental errors are too large to allow a determination of γ.

We define the CP asymmetries as follows:

$$C_{\pi\pi} \equiv \frac{\left(|A|^2 - |\overline{A}|^2\right)}{\left(|A|^2 + |\overline{A}|^2\right)} = \frac{a_{dir}}{B}\,, \qquad S_{\pi\pi} \equiv \frac{2\,\mathrm{Im}\left(e^{-2i\beta}A^*\overline{A}\right)}{\left(|A|^2 + |\overline{A}|^2\right)} = \frac{a_I}{B}\,. \tag{6.1}$$

In terms of these quantities, eq. (2.4) can be written

$$S_{\pi\pi}^2 + C_{\pi\pi}^2 + D_{\pi\pi}^2 = 1\,, \tag{6.2}$$

where $D_{\pi\pi} \equiv a_R/B$ [32]. Then eq. (2.5) becomes

$$\frac{\mathcal{A}_{ct}^2}{B} = \frac{D_{\pi\pi}\cos(2\beta+2\gamma) - S_{\pi\pi}\sin(2\beta+2\gamma) - 1}{\cos 2\gamma - 1}\,. \tag{6.3}$$

According to Method I, $\lambda\mathcal{A}'_{ct}$ is related to $\mathcal{A}_{ct}$. As shown earlier, the leading-order SU(3)-breaking effect is expressible in terms of decay constants only:

$$\frac{\lambda\mathcal{A}'_{ct}}{\mathcal{A}_{ct}} = \frac{f_K}{f_\pi} = 1.22\,. \tag{6.4}$$

We now expand $\cos(2\beta+2\gamma)$ and $\sin(2\beta+2\gamma)$ in eq. (6.3), and write $\cos 2\gamma = \sqrt{1-\sin^2 2\gamma}$. This gives a quadratic equation for $\sin 2\gamma$:

$$\tilde{A}\sin^2 2\gamma + \tilde{B}\sin 2\gamma + \tilde{C} = 0\,, \tag{6.5}$$

where

$$\begin{aligned}
\tilde{A} &= (D_{\pi\pi}\sin 2\beta + S_{\pi\pi}\cos 2\beta)^2 + \left(D_{\pi\pi}\cos 2\beta - S_{\pi\pi}\sin 2\beta - \frac{\mathcal{A}_{ct}^2}{B}\right)^2, \\
\tilde{B} &= 2\,(D_{\pi\pi}\sin 2\beta + S_{\pi\pi}\cos 2\beta)\left(1 - \frac{\mathcal{A}_{ct}^2}{B}\right), \\
\tilde{C} &= \left(1 - \frac{\mathcal{A}_{ct}^2}{B}\right)^2 - \left(D_{\pi\pi}\cos 2\beta - S_{\pi\pi}\sin 2\beta - \frac{\mathcal{A}_{ct}^2}{B}\right)^2.
\end{aligned} \tag{6.6}$$

In order to extract γ, we need the results for the latest measurements of the above quantities. According to ref. [1], we have

$$\sin 2\beta = 0.73 \pm 0.05\,, \qquad \cos 2\beta = 0.68 \pm 0.04\,, \tag{6.7}$$

where we have assumed the value of 2β consistent with the SM. (If $\cos 2\beta = -0.68$ is taken, then this would already be evidence for physics beyond the SM.) ref. [1] also gives

$$BR(B_d^0 \to \pi^+\pi^-) = (4.8 \pm 0.5) \times 10^{-6}\,. \tag{6.8}$$

Writing all amplitudes in units of branching ratios of 10^{-6}, with the B_d^0 lifetime, this gives

$$B = \sqrt{4.8 \pm 0.5} = 2.2 \pm 0.1\,. \tag{6.9}$$

For $B^+ \to \pi^+ K^0$, the most recent papers by Belle and BaBar give [33]

$$\begin{aligned}
\text{Belle}: &\quad BR(B^+ \to \pi^+ K^0) = (2.20 \pm 0.19\ (\text{stat}) \pm 0.11\ (\text{syst})) \times 10^{-5}\,, \\
\text{BaBar}: &\quad BR(B^+ \to \pi^+ K^0) = (2.23 \pm 0.17\ (\text{stat}) \pm 0.11\ (\text{syst})) \times 10^{-5}\,.
\end{aligned} \tag{6.10}$$

Using these numbers, we find

$$BR(B^+ \to \pi^+ K^0) = (2.22 \pm 0.15) \times 10^{-5}\,, \tag{6.11}$$

so that

$${\mathcal{A}'_{ct}}^2 = \frac{(22.2 \pm 1.5)}{r_\tau}\,, \tag{6.12}$$

where $r_\tau = \tau_{B^+}/\tau_{B_d^0} = 1.085$ [1]. This gives

$$\frac{\mathcal{A}_{ct}^2}{B} = \frac{\lambda^2}{B}\left(\frac{f_\pi}{f_K}\right)^2 {\mathcal{A}'_{ct}}^2 = 0.31 \pm 0.03\,. \tag{6.13}$$

It is here that the theoretical error should be added. This error comes from the neglect of small diagrams, as well as SU(3) breaking. For the moment, we do not include any theoretical error; we comment on this below.

For the CP asymmetries, we average the latest Belle and BaBar data [34]:

$$\begin{aligned} S_{\pi\pi} &= -0.70 \pm 0.19\,, \\ C_{\pi\pi} &= -0.42 \pm 0.13\,, \end{aligned} \tag{6.14}$$

giving a value

$$D_{\pi\pi} = \pm(0.58 \pm 0.24)\,. \tag{6.15}$$

Taking the central values of the above numbers, we find:

$$\begin{aligned} D_{\pi\pi} > 0: &\quad \sin 2\gamma = 0.10 \pm 0.57i\,, \\ D_{\pi\pi} < 0: &\quad \sin 2\gamma = 0.87, 0.59\,. \end{aligned} \tag{6.16}$$

A real value for $\sin 2\gamma$ is found only for $D_{\pi\pi} < 0$.

However, we must include the errors on the various measurements. Considering only experimental errors, we find (for $D_{\pi\pi} < 0$)

$$\sin 2\gamma = 0.87 \pm 1.45\,, \qquad 0.59 \pm 1.42\,. \tag{6.17}$$

Recall that a value for $\sin 2\gamma$ gives four values for γ. Thus, it is clear that, within this method, present measurements place very few constraints on the value of γ.

We now turn to the question of theoretical error. In relating the penguin amplitudes for $B_d^0 \to \pi^+\pi^-$ and $B_u^+ \to K^0\pi^+$, we have neglected two small diagrams: PA and P_{EW}^C (or $P_{EW}^{\prime C}$). Each of these is expected to be $\sim 5\%$ of the dominant P (or P') amplitude. (We have also neglected the annihilation diagram A', but this is expected to be tiny compared to P' (eq. (3.5)).) The leading-order SU(3)-breaking term is known with little error (f_π/f_K), but the second-order effect is unknown. The size of this effect is estimated to be $\sim 5\%$. Thus, the total theoretical error in relating $\mathcal{A}_{ct}$ to $\mathcal{A}'_{ct}$ is between 8% (errors added in quadrature) and 15% (errors added linearly). Since the method involves the squares of the amplitudes, the theoretical error in eq. (6.13) is between 15% and 30%.

What is the effect on the extraction of $\sin 2\gamma$? Setting all experimental errors to zero, we consider a theoretical error of 20%. In this case we find

$$\sin 2\gamma = 0.87 \pm 0.50\,, \qquad 0.59 \pm 0.50\,. \tag{6.18}$$

The effect on $\sin 2\gamma$ is obviously very large. The reason is that $\sin 2\gamma$ depends on $\tilde{B}^2 - 4\tilde{A}\tilde{C}$ (see eq. (6.5)). For the particular values of the measurements of $B_d^0 \to \pi^+\pi^-$ and $B_u^+ \to K^0\pi^+$, it turns out that there is a large cancellation between $\tilde{B}^2$ and $4\tilde{A}\tilde{C}$, so that the percentage error here is enormous. This leads to the large error on $\sin 2\gamma$.

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The lesson here is that, although the theoretical error in relating $\mathcal{A}_{ct}$ and $\mathcal{A}'_{ct}$ is indeed between 5% and 15% for all decay pairs, its effect on the extraction of CP phases cannot be predicted. It depends sensitively on the values found for the various measurements of the B decay pairs.

7. Conclusions

We have examined in detail a method for obtaining CP phase information from pairs of B decays. The method works as follows. Consider a decay $B^0 \to M_1 M_2$ which involves a $\bar{b} \to \bar{d}$ penguin amplitude, and in which both B^0 and $\overline{B}^0$ mesons can decay to $M_1 M_2$. Now consider a second decay $B' \to M'_1 M'_2$, related to the first by flavor SU(3), which receives a significant $\bar{b} \to \bar{s}$ penguin contribution and which is dominated by a single amplitude. Using the fact that the two decays are related, one has enough information to extract CP phases from the measurements of the two decays. Depending on how one parametrizes the decay amplitudes, either α or γ can be obtained (assuming that β is independently measured).

There are two sources of theoretical error in this method. First, it may be necessary to neglect certain diagrams in order to find a decay pair related by SU(3). We neglect only amplitudes which are expected to be small (e.g. exchange, annihilation, color-suppressed electroweak penguins); only pairs for which this error is at most 10% are considered. Second, SU(3) breaking, which is typically $O(m_s/\Lambda_{QCD}) \sim 25\%$, must be taken into account. We show that, in general, one can eliminate the leading-order SU(3)-breaking effects, leaving only a second-order error $\sim 5\%$. This occurs in one of two ways. First, for certain decay pairs the leading-order effect depends only on (known) decay constants. Second, for those decays for which this effect depends on (unknown) form factors, there are several ways to proceed. One can measure the ratio of form factors, either in B or in D decays. Alternatively, by taking a double ratio of related SU(3) decays, the leading-order SU(3)-breaking effect cancels.

Our results are shown in table 1. We find twelve decay pairs to which this method can be applied. Some of these — $B_d^0 \to D^+D^-$ and $B_d^0 \to D_s^+D^-$, $B_d^0 \to D^+D^-$ and $B_s^0 \to D_s^+D_s^-$, $B_d^0 \to \pi^+\pi^-$ and $B_u^+ \to K^0\pi^+$, and $B_d^0 \to K^0\bar{K}^0$ and $B_s^0 \to K^0\bar{K}^0$ — have been examined in the literature. However, the other pairs are new. Depending on which decay pair is used, and how one combines the various theoretical uncertainties, the total theoretical error in relating the amplitudes for the decays $B^0 \to M_1M_2$ and $B' \to M'_1M'_2$ is between 5% and 15%. However, the error in the extraction of CP phases cannot be predicted — it depends sensitively on the values of the measurements of the two B decays. The most promising decay pairs are $B_d^0 \to \pi^+\pi^-$ and $B_u^+ \to K^0\pi^+$, for which data is already available, and $B_d^0 \to D^+D^-$ and $B_d^0 \to D_s^+D^-$ or $B_u^+ \to D_s^+\bar{D}^0$. CP-violating measurements of these latter decays will probably be made very soon.

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